Modelling in mechanics 8A

1 a i $x = 0$ gives $h = 0.36 \text{ x } 0 - 0.003 \text{ x } (0)^2 = 0$

Height = 0 m

ii $x = 100$ m gives $h = 0.36 \times 100 - 0.003 \times (100)^2 = 36 - 30$

Height = 6 m

b $x = 200$ m gives $h = 0.36 \times 200 - 0.003 \times (200)^2 = 72 - 120$

Height = −48 m

c The model is not valid for this distance as it predicts the ball would be 48 m below ground level: the ball has already hit the ground at this point.

2 a 90 m (as this is the height when $t = 0$)

b i $t = 3$ gives $h = -5 \times (3)^2 + 15 \times 3 + 90 = -45 + 45 + 90$

Height above sea level = 90 m

ii $t = 5$ gives $h = -5 \times (5)^2 + 15 \times 5 + 90 = -125 + 75 + 90$

Height above sea level = 40 m

c $t = 20$ gives $h = -5 \times (20)^2 + 15 \times 20 + 90 = -2000 + 300 + 90 = -1610$

Height = 1610 m below sea level

d The prediction is incorrect because this height is below sea level, where the model is probably no longer valid, because forces acting on the ball will be different.

3 a When $h = 4$,

$4 = 2 + 1.1x - 0.1x^2$ or rearranging, $0 = -0.1x^2 + 1.1x - 2$, so using the quadratic formula,

$$x = \frac{-(1.1) \pm \sqrt{(1.1)^2 - 4 \times (-0.1) \times (-2)}}{2 \times (-0.1)}$$

$$= \frac{-1.1 \pm \sqrt{0.41}}{-0.2}$$

$$\approx \frac{-1.1 \pm 0.6403}{-0.2}$$

So $x = 2.30$ or 8.70 (to 3 s.f.)

The ball is 4 m above the ground after it has travelled both 2.30 m and 8.70 m horizontally.

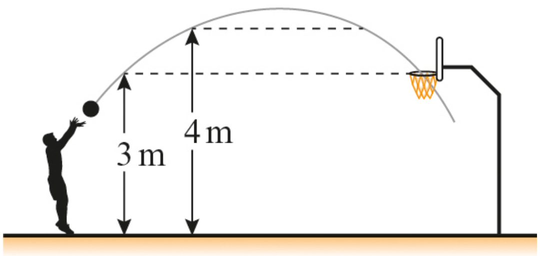
3 b When $h = 3$,

$3 = 2 + 1.1x - 0.1x^2$ or rearranging, $0 = -0.1x^2 + 1.1x - 1$, and dividing by -0.1,

$0 = x^2 - 11x + 10 = (x - 1)(x - 10)$

So $x = 1$ or 10, and the ball is at height 3 m after it has travelled both 1 m and 10 m horizontally.

At the shorter distance, the ball will be travelling upward (see diagram), so $k = 10$.



c If $x > 10$ m the equation is no longer valid as the ball will have gone through (or past) the net, and there would possibly be new forces acting on the ball.

4 a When $t = 1$, $d = 13.2$

So, substituting, $d = kt^2$ becomes $13.2 = k \times 1^2$ and therefore $k = 13.2$.

Our completed equation is $d = 13.2t^2$. When $t = 10$,

$d = 13.2 \times 10^2 = 1320$

The distance travelled is 1320 m.

b Clearly, the model is valid for positive values of t only. We are also unsure of what happens after t = 10, and we therefore can't use this model past that point. The model is valid for $0 \leq t \leq 10$.

5 $h \geq 0$ means that $0.36x - 0.003x^2 = x(0.36 - 0.003x) \geq 0$

We are assuming that $x \geq 0$ after the ball is struck, so we need the bracket to be non-negative:

$0.36 \geq 0.003x$

$$x \leq \frac{0.36}{0.003} = 120$$

So the model is valid for $0 \leq x \leq 120$.

6 When the stone enters the sea, $h = 0$

$0 = -5t^2 + 15t + 90$, or, dividing by -5, $0 = t^2 - 3t - 18 = (t - 6)(t + 3)$

So, $t = -3$ or 6 and the stone hits the sea 6 seconds after it is thrown, since the model is valid only for the time **after** the stone is thrown at t = 0.

Therefore the model is valid for $0 \leq t \leq 6$.

Modelling in mechanics 8B

1 a Modelling the ball as a particle means we can ignore the rotational effect of any external forces that are acting on it and the effects of air resistance, and assume all the mass acts at a single point.

b Assuming air resistance is negligible means we can ignore the frictional effects of the air on the football.

2 a Modelling the puck as a particle means we can ignore the rotational effect of any external forces that are acting on it and the effects of air resistance, and assume all the mass acts at a single point.

b Modelling the ice as smooth means we can ignore any friction between the puck and the ice.

3 Modelling an object as a particle means that the effect of air resistance is ignored but, for a parachute, this force is significant.

4 a Modelling the fishing rod as a light rod means we can assume it has no mass or thickness and is rigid and unbending.

b While the mass of the rod may be negligible in comparison with the reel or any fish it is designed to catch (justifying the 'light' assumption), and narrow compared to its length (allowing it to be treated as a one-dimensional object) rigidity is not a desirable property of fishing rods, so it is not appropriate to consider it as a rod.

5 a Model the golf ball as a particle, and ignore the effects of air resistance.

b Model the child and sledge as a single particle, consider the hill to be smooth, and ignore the effects of air resistance.

c Model the objects as particles, the string as light and inextensible, and the pulley as smooth.

d Model the suitcase and handle as a single particle, consider the path to be smooth, and ignore friction between the wheels and their holdings.

Modelling in mechanics 8C

1 **a** $65 \text{ km h}^{-1} = \frac{65\times1000}{60\times60} \text{ m s}^{-1} = 18.1 \text{ m s}^{-1}$ (to 3 s.f.)

b $15 \text{ g cm}^{-2} = \frac{15\div1000}{1\div(100\times100)} \text{ kg m}^{-2} = 150 \text{ kg m}^{-2}$

c $30 \text{ cm per minute} = \frac{30\div100}{60} \text{ m s}^{-1} = 5\times10^{-3} \text{ m s}^{-1}$

d $24 \text{ g m}^{-3} = \frac{24}{1000} \text{ kg m}^{-3} = 2.4\times10^{-2} \text{ kg m}^{-3}$

e $4.5\times10^{-2} \text{ g cm}^{-3} = \frac{4.5\times10^{-2}\div1000}{1\div(100\times100\times100)} \text{ kg m}^{-3} = 45 \text{ kg m}^{-3}$

f $6.3\times10^{-3} \text{ kg cm}^{-2} = \frac{6.3\times10^{-3}}{1\div(100\times100)} \text{ kg m}^{-2} = 63 \text{ kg m}^{-2}$

2 **a** A: normal reaction, B: forward thrust, C: weight, D: friction

b A: buoyancy, B: forward thrust, C: weight, D: drag/water resistance

c A: normal reaction, B: friction, C: weight, D: tension

d A: normal reaction, B: weight, C: friction

Modelling in mechanics 8D

1 **a** 2.1 m s^{-1}

b 500 m

c -1.8 m s^{-1}

d -2.7 m s^{-1}

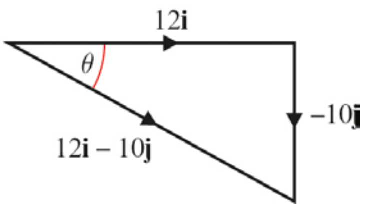
e -750 m

f 2.5 m s^{-1}

2 **a** speed $|\mathbf{v}| = \sqrt{12^2 + 10^2} = \sqrt{244}$

The speed of the car is 15.6 m s^{-1} (to 3 s.f.)

b Let the acute angle made with **i** be θ, then



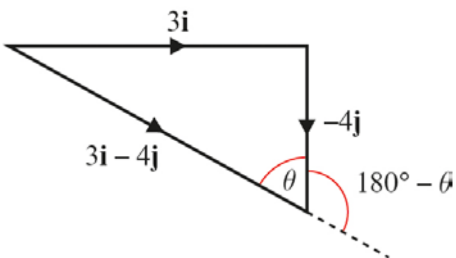
$$\tan\theta = \frac{10}{12} = 0.8333 \text{ so } \theta = 39.8° \text{ (to 3 s.f.)}$$

The direction of motion of the car is 39.8° from the **i** vector.

3 **a** $|\mathbf{a}| = \sqrt{3^2 + 4^2} = \sqrt{25}$

The magnitude of the acceleration is 5 m s^{-2}.

b Let the acute angle made with **j** be θ



$$\tan\theta = \frac{3}{4} = 0.75 \text{ so } \theta = 36.9° \text{ (to 3 s.f.)}$$

Angle required = 180° – θ = 180° – 36.9° = 143.1°

The direction of the acceleration is 143° from the **j** vector.

4 a $\overrightarrow{AC} = \overrightarrow{AB} + \overrightarrow{BC}$

$$\overrightarrow{AC} = \begin{pmatrix} 10 \\ 3 \end{pmatrix} + \begin{pmatrix} -7 \\ 12 \end{pmatrix} = \begin{pmatrix} 3 \\ 15 \end{pmatrix}$$

$$\left|\overrightarrow{AC}\right| = \sqrt{3^2 + 15^2} = \sqrt{234}$$

The magnitude of the displacement is 15.3 m (to 3 s.f.)

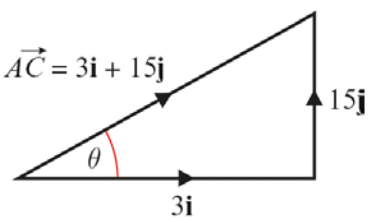
b $\left|\overrightarrow{AB}\right| = \sqrt{10^2 + 3^2} = \sqrt{109}$

$$\left|\overrightarrow{BC}\right| = \sqrt{7^2 + 12^2} = \sqrt{193}$$

$$\left|\overrightarrow{AB}\right| + \left|\overrightarrow{BC}\right| = \sqrt{109} + \sqrt{193} = 24.3$$

The girl cycles 24.3 km (to 3 s.f.)

c Let the acute angle made with **i** be θ



$$\tan\theta = \frac{15}{3} = 5 \text{ so } \theta = 78.7° \text{ (to 3 s.f.)}$$

The direction of motion of the car is 78.7° from the **i** vector.

Modelling in mechanics, Mixed Exercise 8

1 a $x = 2$ gives

$$h = \frac{1}{10}\left(24 \times 2 - 3 \times (2)^2\right)$$
$$= \frac{1}{10}\left(48 - 12\right) = 3.6$$

When it is 2 m horizontally from where it is hit, the ball is at a vertical height of 3.6 m.

b When $h = 2.1$,

$2.1 = \frac{1}{10}\left(24x - 3x^2\right)$ or rearranging, $0 = 24x - 21 - 3x^2$, and dividing by -3,

$0 = x^2 - 8x + 7 = (x - 1)(x - 7)$

So $x = 1$ or 7.

The ball is at a height of 2.1m when it is at a horizontal distance of 1 m and again at 7 m.

c Model becomes valid when the ball is hit, i.e. $x = 0$.

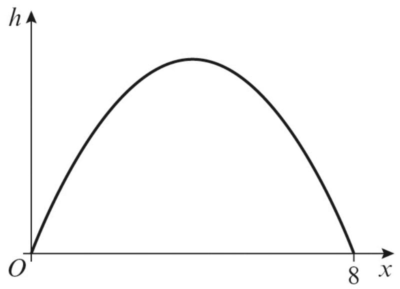
The model remains valid while the ball is above the ground, i.e. $h \geqslant 0$

$\frac{1}{10}\left(24x - 3x^2\right) \geq 0$ or factorising, $0.3x(8 - x) \geq 0$

We know that $x \geq 0$ for our region, so we need the bracket to be non-negative, i.e. $x \leq 8$.

The model is valid for $0 \leq x \leq 8$.

d The equation for this model produces a curve of the form shown:



Since the curve is symmetrical, maximum height occurs when $x = 4$, at which point:

$$h = \frac{1}{10}\left(24 \times 4 - 3 \times (4)^2\right)$$
$$= \frac{1}{10}\left(96 - 48\right) = 4.8$$

The maximum height of the ball is 4.8 m.

2 a $x = 2$ gives

$h = 10 - 0.58 \times (2)^2 = 7.68$

When the horizontal distance from the end of the board is 2 m, the diver is at a height of 7.68 m.

b When the diver hits the water, $h = 0$

$0 = 10 - 0.58x^2$

$0.58x^2 = 10$

$$x = \sqrt{\frac{10}{0.58}} \approx \sqrt{17.24}$$

The diver enters the water 4.15 m from the end of the board.

c By modelling the diver as a particle, we can ignore air resistance and the rotational effects of external forces. The mass of the diver is assumed to be concentrated at a single point.

d The pool is only 4.5 m deep so the model could not be accurate for large values of x.
Also, the motion is likely to be different in the water than in the air.

3 a Model the man on skis as a particle. This allows one to ignore the rotational effect of any forces that are acting on the man as well as any effects due to air resistance.

Consider the slope to be smooth: that there is no friction between the skis and the slope.

b Model the yo-yo as a particle. This allows one to ignore the rotation of the yo-yo and air resistance. We assume that the mass of the yo-yo is concentrated at a point.

Consider the string to be light and inextensible. This allows one to ignore the weight of the string and assume it does not stretch, thereby affecting the acceleration of attached objects.

Model the yo-yo as smooth, that is, assume there is no friction between the yo-yo and the string.

4 a 2.5 km per minute $= \dfrac{2.5 \times 1000}{60}$ m s^{-1} $= 41.7$ m s^{-1} (to 3 s.f.)

b $0.6 \text{ kg cm}^{-2} = \dfrac{0.6}{1 \div (100 \times 100)} \text{ kg m}^{-2} = 6000 \text{ kg m}^{-2}$

c $1.2 \times 10^3 \text{ g cm}^{-3} = \dfrac{1.2 \times 10^3 \times (100 \times 100 \times 100)}{1000} \text{ kg m}^{-3} = 1.2 \times 10^6 \text{ kg m}^{-3}$

5 a Model the ball as a particle.

Assume the floor is smooth.

5 b i The velocity will be positive as the positive direction is defined as such.

ii In real life the acceleration would be negative, as the ball always slows down. However, if we assume there is no friction, then the ball would move at constant velocity.

6 a Velocity is positive, displacement is positive

b Velocity is negative, displacement is positive

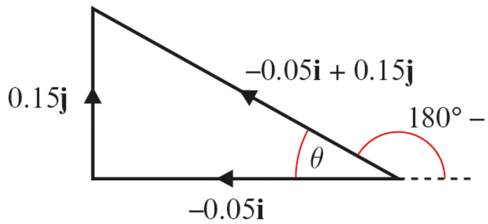
c Velocity is negative, displacement is negative

7 a $|\mathbf{a}| = \sqrt{0.05^2 + 0.15^2} = \sqrt{0.025} = 0.158$ (to 3 s.f.)

The magnitude of the acceleration is 0.158 m s^{-2}.

b Let the acute angle made with **i** be θ, then

$\tan\theta = \frac{0.15}{0.05} = 3$ so $\theta = 71.6°$ (to 3 s.f.)



Angle required = 180° − θ = 180° − 71.6° = 108° (to 3 s.f.)

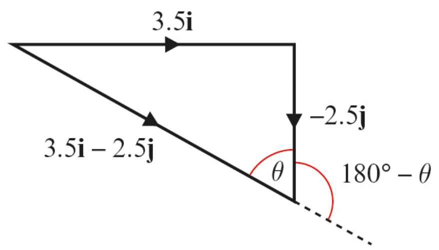
The direction of the acceleration is 108° from the **i** vector.

8 a $|\mathbf{v}| = \sqrt{3.5^2 + 2.5^2} = \sqrt{18.5} = 4.30$ (to 3 s.f.)

The speed of the toy car is 4.30 m s^{-1}.

b Let the acute angle made with **j** be θ, then

$\tan\theta = \frac{3.5}{2.5} = 1.4$ so $\theta = 54.5°$ (to 3 s.f.)



Angle required = 180° − θ = 180° − 54.5° = 126° (to 3 s.f.)

The direction of the acceleration is 126° from the **j** vector.

9 a $\overrightarrow{PR} = \overrightarrow{PQ} + \overrightarrow{QR}$

$\overrightarrow{PR} = \begin{pmatrix} 100 \\ 80 \end{pmatrix} + \begin{pmatrix} 50 \\ -30 \end{pmatrix} = \begin{pmatrix} 150 \\ 50 \end{pmatrix}$

$\left|\overrightarrow{PR}\right| = \sqrt{150^2 + 50^2} = \sqrt{25000} = 158$ (to 3 s.f.)

The magnitude of the displacement is 158 m.

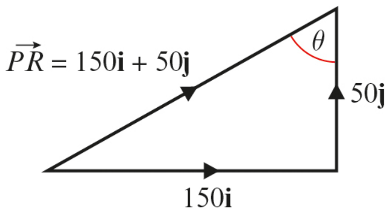
b $\left|\overrightarrow{PQ}\right| = \sqrt{100^2 + 80^2} = \sqrt{16400} = 128$ (to 3 s.f.)

$\left|\overrightarrow{QR}\right| = \sqrt{50^2 + 30^2} = \sqrt{3400} = 58.3$ (to 3 s.f.)

$\left|\overrightarrow{PQ}\right| + \left|\overrightarrow{QR}\right| = \sqrt{16400} + \sqrt{3400} = 186$ (to 3 s.f.)

The plane travels a total distance of 186 m.

c Let the acute angle made with **j** be θ



$\tan\theta = \frac{150}{50} = 3$ so $\theta = 71.6°$ (to 3 s.f.)

The direction of motion of the car is 71.6° from the **j** vector.

Constant acceleration 9A

1 a *A* displacement = 40 km, time = 0.5 h and $\frac{40}{0.5}=80$

So the average velocity is 80 km h^{-1}.

B displacement = 20 km, time = 0.5 h and $\frac{20}{0.5}=40$

So the average velocity is 40 km h^{-1}.

C displacement = 0 km, time = 0.5 h and $\frac{0}{0.5}=0$

So the average velocity is 0 km h^{-1}.

D displacement = 40 km, time = 1 h and $\frac{40}{1}=40$

So the average velocity is 40 km h^{-1}.

E displacement = −100 km, time = 1.5 h and $\frac{100}{1.5}=-66.7$ (to 3 s.f.)

So the average velocity is -66.7 km h^{-1}.

b The average velocity for the whole journey is 0 km h^{-1} as the overall displacement is 0 km.

c Total distance travelled = 200 km
Total time taken = 4 h

average speed = $\frac{200}{4}=50\text{ km h}^{-1}$

2 a For first section of the journey: average velocity = 60 km h^{-1}, time taken = 2.5 h

displacement = 2.5 × 60 = 150 km

This is 6 squares on the vertical axis, so one square is $\frac{150}{6}=25\text{ km}$

total displacement shows as 7.5 squares = 7.5 × 25 = 187.5 km

b Time for whole journey = 3.75 h

average velocity = $\frac{187.5}{3.75}=50\text{ km h}^{-1}$

3 a displacement = 12 km, time = 1 h

average velocity = $\frac{12}{1}=12\text{ km h}^{-1}$

b Sarah passed her home at 12:45.

c For the penultimate stage: displacement = −12 + (−3) = −15 km, time = 1.5 h

average velocity = $\frac{-15}{1.5}=-10\text{ km h}^{-1}$

For the final stage: displacement = 3 km, time = 1 h

average velocity = $\frac{3}{1}=3\text{ km h}^{-1}$

3 d Total distance travelled = 30 km
Total time taken = 4 h

average speed = $\frac{30}{4} = 7.5$ km h^{-1}

4 a Reading from the graph:

maximum height = 2.5 m
time taken to reach this = 0.75 s

b When it reaches the highest point, the velocity of the ball is 0 m s^{-1}.

c i The velocity of the ball is positive (upwards) and decreases (the ball is decelerating) until it reaches 0 at the highest point.

ii The velocity of the ball is negative (downwards), and increases (the ball is accelerating) until it hits the ground at the same speed at which it was launched.

Constant acceleration 9B

1 a $a = \frac{9}{4} = 2.25$

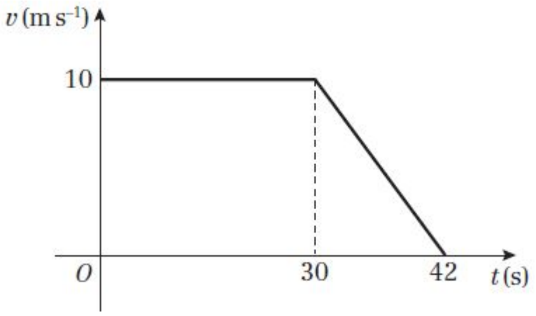
The athlete accelerates at a rate of 2.25 m s^{-2}.

b $s = \frac{1}{2}(a+b)h$

$= \frac{1}{2}(8+12)\times 9 = 90$

The displacement of the athlete after 12 s is 90 m.

2 a



b $s = \frac{1}{2}(a+b)h$

$= \frac{1}{2}(30+42)\times 10 = 360$

The distance from A to B is 360 m.

3 a $a = \frac{8}{20} = 0.4$

The acceleration of the cyclist is 0.4 m s^{-2}.

b $a = -\frac{8}{15} = -0.533$ (to 3 s.f.)

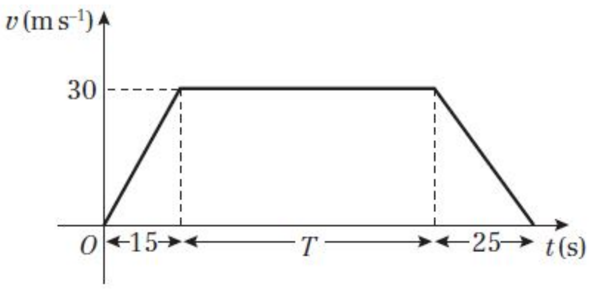
The deceleration of the cyclist is 0.533 m s^{-2}.

c $s = \frac{1}{2}(a+b)h$

$= \frac{1}{2}(40+75)\times 8 = 460$

After 75 s, the distance from the starting point of the cyclist is 460 m.

4 a



4 b $s = \frac{1}{2}(a+b)h$

$2400 = \frac{1}{2}(T + (15 + T + 25)) \times 30$

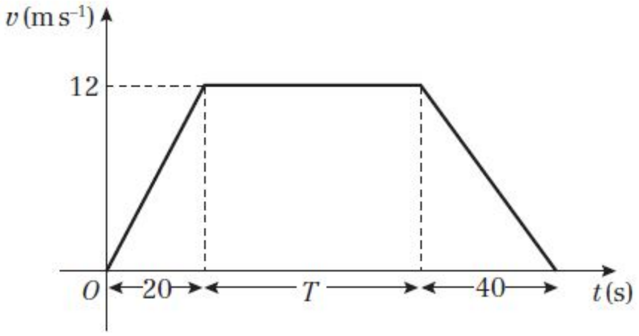
$= 15(2T + 40)$

$2T + 40 = \frac{2400}{15} = 160$

$T = \frac{160 - 40}{2} = 60$

The time taken to travel from S to F is $(15 + T + 25) = 100$ s.

5 a The velocity after 20 s is given by



velocity = acceleration x time $= 0.6 \times 20 = 12$

b $s = \frac{1}{2}(a+b)h$

$4200 = \frac{1}{2}(T + (20 + T + 40)) \times 12$

$= 6(2T + 60)$

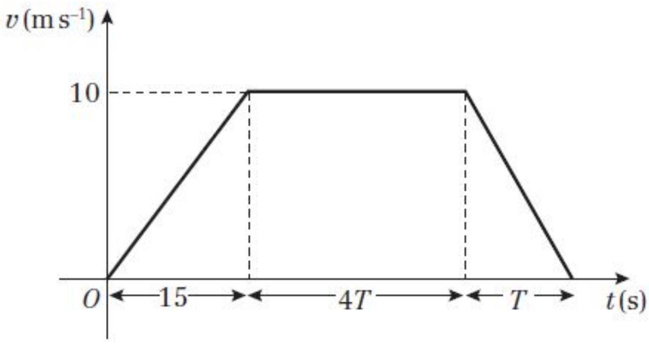
$2T + 60 = \frac{4200}{6} = 700$

$T = \frac{700 - 60}{2} = 320$

c While at constant velocity: $v = 12$ m s^{-1}, $t = 320$ s

distance travelled = 12 × 320 = 3840 m

6 a



6 b $s = \frac{1}{2}(a+b)h$

$$480 = \frac{1}{2}(4T + (15 + 4T + T))10$$
$$= 5 \times (15 + 9T)$$
$$9T + 15 = \frac{480}{5} = 96$$

$$T = \frac{96-15}{9} = 9$$

Total time travelling = 15 + 5T = 15 + (5 × 9) = 60

The particle travels for a total of 60 s.

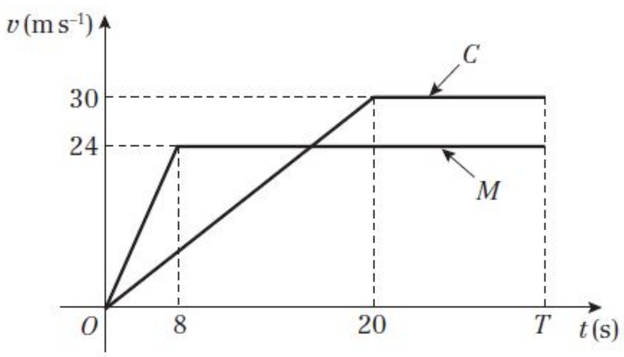
7 a Area = trapezium + rectangle + triangle

$$100 = \frac{1}{2}(u+10) \times 3 + 7 \times 10 + \frac{1}{2} \times 2 \times 10$$
$$= \frac{3}{2}(u+10) + 70 + 10$$
$$\frac{3}{2}(u+10) = 100 - 70 - 10 = 20$$
$$u = 20 \times \frac{2}{3} - 10$$
$$= \frac{10}{3}$$

b $a = \dfrac{10 - \frac{10}{3}}{3} = \frac{20}{9} = 2.22$ (to 3 s.f.)

The acceleration of the particle is 2.22 m s^{-2}.

8 a For M, velocity = acceleration x time = $3 \times 8 = 24$



b Let C overtake M at time T seconds.

The distance travelled by M is given by

$$s = \frac{1}{2}(8 \times 24) + 24 \times (T - 8)$$
$$= 24(T - 4)$$

8 b The distance travelled by C is given by

$$s = \tfrac{1}{2}(a+b)h = \tfrac{1}{2}(T-20+T)\times 30$$
$$= 15\,(2T-20)$$

At the point of overtaking the distances are equal.

$$24(T-4) = 15(2T-20)$$
$$24T-96 = 30T-300$$
$$6T = 204$$
$$T = \tfrac{204}{6} = 34$$

$$s = 24(T-4)$$
$$= 24(34-4) = 720$$

The distance of the pedestrian from the road junction is 720 m.

Challenge

a The object changed direction after 6 s, as this is when the velocity changed from positive to negative.

b While travelling at positive velocity:

$$s_p = \tfrac{1}{2}(1+6)\times 3 = \tfrac{1}{2}\times 21 = 10.5$$

While travelling at negative velocity:

$$s_n = \tfrac{1}{2}(4+2)\times 2 = \tfrac{1}{2}\times 12 = 6$$

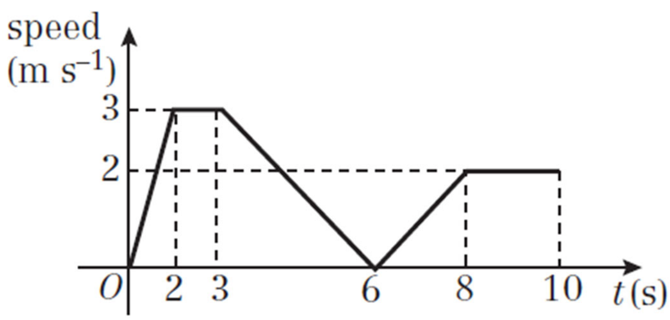
The total distance travelled by the object = $s_p + s_n$ = 10.5 + 6 = 16.5 m

c i Using the value calculated in **b**, after 6 s the displacement of the object is s_p = 10.5 m.

ii In the first 6 seconds, displacement is positive.
In the last 4 seconds, displacement is negative.

Hence, using the values calculated in **b**, total displacement = $s_p + (-s_n)$ = 10.5 + (−6) = 4.5 m.

d



Constant acceleration 9C

1 $a = 3, u = 2, t = 6, v = ?$

$$v = u + at$$
$$= 2 + 3 \times 6 = 2 + 18 = 20$$

The velocity of the particle at time $t = 6$ s is 20 m s^{-1}.

2 $u = 10, v = 0, t = 16, a = ?$

$$v = u + at$$

$$0 = 10 + a \times 16$$
$$a = -\tfrac{10}{16} = -0.625$$

The deceleration of the car is 0.625 m s^{-1}.

3 $s = 360, t = 15, v = 28, u = ?$

$$s = \left(\frac{u+v}{2}\right)t$$
$$360 = \frac{u+28}{2} \times 15$$
$$u = \frac{360 \times 2}{15} - 28$$
$$= 20$$

The velocity of the car at the first sign post is 20 m s^{-1}.

4 a $a = 0.5, u = 3, t = 12, v = ?$

$$v = u + at$$
$$= 3 + 0.5 \times 12 = 3 + 6 = 9$$

The velocity of the cyclist at B is 9 m s^{-1}.

b $u = 3, v = 9, t = 12, s = ?$

$$s = \left(\frac{u+v}{2}\right)t$$
$$= \left(\frac{3+9}{2}\right) \times 12 = 6 \times 12 = 72$$

The distance from A to B is 72 m.

5 a $s = 24, t = 6, v = 5, u = ?$

$$s = \left(\frac{u+v}{2}\right)t$$

$$24 = \left(\frac{u+5}{2}\right)\times 6$$

$$u = \frac{24\times 2}{6} - 5 = 3$$

The velocity of the particle at A is 3 m s^{-1}.

b $u = 3, v = 5, t = 6, a = ?$

$$v = u + at$$
$$5 = 3 + 6a$$
$$a = \frac{5-3}{6} = \frac{1}{3} = 0.333 \text{ (to 3 s.f.)}$$

The acceleration of the particle is 0.333 m s^{-2}.

6 a $a = -1.2, t = 6, v = 2, u = ?$

$$v = u + at$$
$$2 = u - 1.2\times 6 = u - 7.2$$
$$u = 2 + 7.2 = 9.2$$

The speed of the particle at A is 9.2 m s^{-1}.

b $u = 9.2, v = 2, t = 6, s = ?$

$$s = \left(\frac{u+v}{2}\right)t = \left(\frac{9.2+2}{2}\right)\times 6 = 11.2\times 3 = 33.6$$

The distance from A to B is 33.6 m.

7 a $72 \text{ km h}^{-1} = 72\times 1000 \text{ m h}^{-1} = \frac{72\times 1000}{3600} \text{ m s}^{-1} = 20 \text{ m s}^{-1}$

$u = 20, a = -0.6, t = 25, v = ?$

$$v = u + at = 20 - 0.6\times 25 = 20 - 15 = 5 \text{ m s}^{-1}$$

$$5 \text{ m s}^{-1} = \frac{5\times 3600}{1000} \text{ km h}^{-1} = 18 \text{ km h}^{-1}$$

The speed of the train as it passes the second signal is 18 km h^{-1}

b $u = 20, v = 5, t = 25, s = ?$

$$s = \left(\frac{u+v}{2}\right)t = \left(\frac{20+5}{2}\right)\times 25 = 12.5\times 25 = 312.5$$

The distance between the signals is 312.5 m.

8 a $a = -4, u = 32, v = 0, t = ?$

$$v = u + at$$
$$0 = 32 - 4t$$
$$t = \frac{32}{4} = 8$$

The time taken for the particle to move from A to B is 8 s.

b $u = 32, v = 0, t = 8, s = ?$

$$s = \left(\frac{u+v}{2}\right)t = \left(\frac{32+0}{2}\right) \times 8 = 16 \times 8 = 128$$

The time between A and B is 128 m.

9 a $u = 16,\ t = 40,\ v = 0,\ a = ?$

$$v = u + at$$
$$0 = 16 + 40a$$
$$a = \frac{-16}{40} = -0.4$$

The deceleration between A and B is 0.4 m s^{-2}.

b $u = 16, t = 40, v = 0,\ s = ?$

$$s = \left(\frac{u+v}{2}\right)t = \left(\frac{16+0}{2}\right) \times 40 = 8 \times 40 = 320$$

The distance from the bottom of the hill to the point where the skier comes to rest is 320 m.

10 a $u = 2, v = 7, t = 20, a = ?$

$$v = u + at$$
$$7 = 2 + 20a$$
$$a = \frac{7-2}{20} = 0.25$$

The acceleration of the particle is 0.25 m s^{-2}.

b From B to C, $u = 7,\ v = 11,\ a = 0.25,\ t = ?$

$$v = u + at$$
$$11 = 7 + 0.25t$$
$$t = \frac{11-7}{0.25} = 16$$

10 b The time taken for the particle to move from *B* to *C* is 16 s.

c The time taken to move from *A* to *C* is (20 + 16) = 36 s

From *A* to *C*, $u = 2,\ v = 11,\ t = 36,\ s = ?$

$$s = \left(\frac{u+v}{2}\right)t = \left(\frac{2+11}{2}\right)\times 36 = 6.5\times 36 = 234$$

The distance between *A* and *C* is 234 m.

11 a From *A* to *B*, $a = 1.5,\ u = 1,\ t = 12,\ v = ?$

$v = u + at = 1 + 1.5\times 12 = 1 + 18 = 19$

The velocity of the particle at *B* is 19 m s^{-1}.

b From *B* to *C*, $u = 19,\ v = 43,\ t = 10,\ a = ?$

$$v = u + at$$
$$43 = 19 + 10a$$
$$a = \frac{43-19}{10} = 2.4$$

The acceleration from *B* to *C* is 2.4 m s^{-2}.

c The distance from *A* to *B*, $u = 1,\ v = 19,\ t = 12,\ s = ?$

$$s = \left(\frac{u+v}{2}\right)t = \left(\frac{1+19}{2}\right)\times 12 = 10\times 12 = 120$$

The distance from *B* to *C*, $u = 19,\ v = 43, t = 10,\ s = ?$

$$s = \left(\frac{u+v}{2}\right)t = \left(\frac{19+43}{2}\right)\times 10 = 31\times 10 = 310$$

The distance from *A* to *C* is (120 + 310) = 430 m.

12 a $u = 0,\ v = 5,\ t = 20,\ a = x$

$$v = u + at$$
$$5 = 0 + 20x$$
$$x = \tfrac{5}{20} = 0.25$$

b While accelerating, $u = 0,\ v = 5,\ t = 20,\ s = ?$

$$s = \left(\frac{u+v}{2}\right)t = \left(\frac{0+5}{2}\right)\times 20 = 2.5\times 20 = 50$$

12 b While decelerating, $u = 5$, $v = 0$, $a = -\frac{1}{2}$, $x = -0.125$, $t = ?$

$$v = u + at$$
$$0 = 5 - 0.125t$$
$$t = \frac{5}{0.125} = 40$$

Now, $u = 5$, $v = 0$, $t = 40$, $s = ?$

$$s = \left(\frac{u+v}{2}\right)t = \left(\frac{5+0}{2}\right) \times 40 = 2.5 \times 40 = 100$$

The total distance travelled is the distance travelled while accelerating added to the distance travelled while decelerating = (50 + 100) = 150 m.

13 a From A to B

$$v = u + at$$
$$30 = 20 + at_1$$
$$at_1 = 10 \qquad \mathbf{(1)}$$

From B to C

$$v = u + at$$
$$45 = 30 + at_2$$
$$at_2 = 15 \qquad \mathbf{(2)}$$

Dividing (**1**) by (**2**),

$$\frac{at_1}{at_2} = \frac{10}{15}$$
$$\frac{t_1}{t_2} = \frac{2}{3} \quad \text{as required.}$$

b From the result in part **a**

$$t_2 = \tfrac{3}{2}t_1$$
$$t_1 + t_2 = t_1 + \tfrac{3}{2}t_1 = \tfrac{5}{2}t_1 = 50$$
$$t_1 = \tfrac{2}{5} \times 50 = 20$$

From A to B, $u = 20$, $v = 30$, $t = 20$, $s = ?$

$$s = \left(\frac{u+v}{2}\right)t = \left(\frac{20+30}{2}\right) \times 20 = 25 \times 20 = 500$$

The distance from A to B is 500 m.

Challenge

a Distance s is the same for both particles: AB.

For the first particle: $u = 3$, $v = 5$, time taken is t seconds

$$s = \left(\frac{u+v}{2}\right)t = \left(\frac{3+5}{2}\right)t = 4t \qquad (\mathbf{1})$$

For the second particle: $u = 4$, $v = 8$, time taken is $(t - 1)$ seconds, because the particle starts 1 second later than the first and arrives at the same time)

$$s = \left(\frac{u+v}{2}\right)(t-1) = \left(\frac{4+8}{2}\right)(t-1) = 6(t-1) = 6t-6 \qquad (\mathbf{2})$$

$4t = 6t - 6$ (**1**) and (**2**)

$t = 3$

The time for the first particle to get from A to B is 3 s.

b Substituting this value of t into equation (**1**):

$s = 4t = 4 \times 3 = 12$

The distance between A and B is 12 m.

[Check by substituting into equation (**2**): $s = 6t - 6 = 6 \times 3 - 6 = 12$]

Constant acceleration 9D

1 $a = 2.5,\ u = 3,\ s = 8,\ v = ?$

$$v^2 = u^2 + 2as = 3^2 + 2 \times 2.5 \times 8 = 9 + 40 = 49$$
$$v = \sqrt{49} = 7$$

The velocity of the particle as it passes through B is $7\,\mathrm{m\,s^{-1}}$.

2 $u = 8,\ t = 6,\ s = 60,\ a = ?$

$$s = ut + \tfrac{1}{2}at^2$$
$$60 = 8 \times 6 + \tfrac{1}{2} \times a \times 6^2 = 48 + 18a$$
$$a = \frac{60 - 48}{18} = \frac{2}{3}$$

The acceleration of the car is $0.667\,\mathrm{m\,s^{-2}}$ (to 3 s.f.)

3 $u = 12,\ v = 0,\ s = 36,\ a = ?$

$$v^2 = u^2 + 2as$$
$$0^2 = 12^2 + 2 \times a \times 36 = 144 + 72a$$
$$a = -\tfrac{144}{72} = -2$$

The deceleration is $2\ \mathrm{m\,s^{-2}}$.

4 $u = 15,\ v = 20,\ s = 500,\ a = ?$ $\quad 54\,\mathrm{km\,h^{-1}} = \dfrac{54 \times 1000}{3600}\,\mathrm{m\,s^{-1}} = 15\,\mathrm{m\,s^{-1}}$

$$72\,\mathrm{km\,h^{-1}} = \frac{72 \times 1000}{3600}\,\mathrm{m\,s^{-1}} = 20\,\mathrm{m\,s^{-1}}$$

$$v^2 = u^2 + 2as$$
$$20^2 = 15^2 + 2 \times a \times 500$$
$$400 = 225 + 1000a$$
$$a = \frac{400 - 225}{1000} = 0.175$$

The acceleration of the train is $0.175\,\mathrm{m\,s^{-2}}$.

5 a $s = 48,\ u = 4,\ v = 16,\ a = ?$

$v^2 = u^2 + 2as$

$16^2 = 4^2 + 2 \times a \times 48$

$256 = 16 + 96a$

$a = \dfrac{256 - 16}{96} = 2.5$

The acceleration of the particle is $2.5\,\text{m s}^{-2}$.

b $u = 4,\ v = 16,\ a = 2.5,\ t = ?$

$v = u + at$

$16 = 4 + 2.5t$

$t = \dfrac{16 - 4}{2.5} = 4.8$

The time taken to move from A to B is 4.8 s.

6 a $a = 3, s = 38, t = 4, u = ?$

$s = ut + \frac{1}{2}at^2$

$38 = 4u + \frac{1}{2} \times 3 \times 4^2 = 4u + 24$

$u = \dfrac{38 - 24}{4} = 3.5$

The initial velocity of the particle is $3.5\,\text{m s}^{-1}$.

b $a = 3, t = 4, u = 3.5, v = ?$

$v = u + at = 3.5 + 3 \times 4 = 15.5$

The final velocity of the particle is $15.5\,\text{m s}^{-1}$.

7 a $u = 18,\ v = 0,\ a = -3,\ s = ?$

$v^2 = u^2 + 2as$

$0^2 = 18^2 + 2 \times (-3) \times s = 324 - 6s$

$s = \frac{324}{6} = 54$

The distance travelled as the car decelerates is 54 m.

7 b $u = 18,\ v = 0,\ a = -3,\ t = ?$

$$v = u + at$$
$$0 = 18 - 3t$$
$$t = \tfrac{18}{3} = 6$$

The time taken for the car to decelerate is 6 s.

8 a $u = 12,\ v = 0,\ a = -0.8,\ s = ?$

$$v^2 = u^2 + 2as$$
$$0^2 = 12^2 + 2 \times (-0.8) \times s = 144 - 1.6s$$
$$s = \tfrac{144}{1.6} = 90$$

The distance moved by the stone is 90 m.

b Half the distance in **a** is 45 m.

$u = 12,\ a = -0.8,\ s = 45,\ v = ?$

$$v^2 = u^2 + 2as$$
$$= 12^2 + 2 \times (-0.8) \times 45 = 144 - 72 = 72$$
$$v = \sqrt{72} = 8.49 \text{ (to 3 s.f.)}$$

The speed of the stone is $8.49\,\text{m}\,\text{s}^{-1}$.

9 a $a = 2.5,\ u = 8,\ s = 40,\ t = ?$

$$s = ut + \tfrac{1}{2}at^2$$
$$40 = 8t + 1.25t^2$$
$$0 = 1.25t^2 + 8t - 40$$
$$t = \frac{-(8) \pm \sqrt{(8)^2 - 4 \times (1.25) \times (-40)}}{2 \times (1.25)}$$
$$t = \frac{-8 + \sqrt{264}}{2.5} = 3.30 \text{ (to 3 s.f.)}$$

The time taken for the particle to move from O to A is 3.30 s.

b $a = 2.5,\ u = 8,\ s = 40,\ v = ?$

$$v^2 = u^2 + 2as$$
$$= 8^2 + 2 \times 2.5 \times 40 = 264$$
$$v = \sqrt{264} = 16.2 \text{ (to 3 s.f.)}$$

The speed of the particle at A is $16.2\,\text{m}\,\text{s}^{-1}$.

10 a $a = -2,\ s = 32,\ u = 12,\ t = ?$

$$s = ut + \tfrac{1}{2}at^2$$
$$32 = 12t - t^2$$
$$t^2 - 12t + 32 = (t-4)(t-8) = 0$$

So $t = 4$ or $t = 8$.

b When $t = 4$,

$$v = u + at = 12 - 2 \times 4 = 4$$

The velocity is $4\,\text{m s}^{-1}$ in the direction $\overrightarrow{AB}$.

When $t = 8$,

$$v = u + at = 12 - 2 \times 8 = -4$$

The velocity is $4\,\text{m s}^{-1}$ in the direction $\overrightarrow{BA}$.

11 a $a = -5,\ u = 12,\ s = 8,\ t = ?$

$$s = ut + \tfrac{1}{2}at^2$$
$$8 = 12t - 2.5t^2$$
$$2.5t^2 - 12t + 8 = 0$$
$$5t^2 - 24t + 16 = (5t-4)(t-4) = 0$$

So $t = 0.8$ or $t = 4$.

b $a = -5,\ u = 12,\ s = -8,\ v = ?$

$$\begin{aligned} v^2 &= u^2 + 2as \\ &= 12^2 + 2 \times (-5) \times (-8) \\ &= 144 + 80 = 224 \\ v &= \sqrt{224} = -15.0 \text{ (to 3 s.f.)} \end{aligned}$$

The velocity at $x = -8$ is $-15.0\,\text{m s}^{-1}$.

12 a $a = -4,\ u = 14,\ s = 22.5,\ t = ?$

$$s = ut + \tfrac{1}{2}at^2$$
$$22.5 = 14t - 2t^2$$
$$2t^2 - 14t + 22.5 = 0$$
$$4t^2 - 28t + 45 = (2t-5)(2t-9) = 0$$

The difference between the times is $(4.5 - 2.5)\text{ s} = 2\text{ s}$.

12 b The maximum distance is reached when P reverses direction.
$a = -4,\ u = 14,\ v = 0,\ t = ?$

$v = u + at$

$0 = 14 - 4t \Rightarrow t = \frac{14}{4} = 3.5$

Find the displacement when $t = 3.5$.

$s = ut + \frac{1}{2}at^2$

$\quad = 14 \times 3.5 - 2 \times 3.5^2 = 24.5$

Between $t = 2.5$ and $t = 4.5$ the particle moves back and forward.

Hence total distance travelled $= 2 \times (24.5 - 22.5)$ m $= 4$ m.

13 a From B to C, $u = 14,\ v = 20,\ s = 300,\ a = ?$

$$v^2 = u^2 + 2as$$
$$20^2 = 14^2 + 2 \times a \times 300$$
$$a = \frac{20^2 - 14^2}{600} = 0.34$$

The acceleration of the car is 0.34 m s^{-2}.

b From A to C, $v = 20,\ s = 400,\ a = 0.34,\ u = ?$

$$v^2 = u^2 + 2as$$
$$20^2 = u^2 + 2 \times 0.34 \times 400 = u^2 + 272$$
$$u^2 = 400 - 272 = 128$$
$$u = \pm\sqrt{128} = \pm 8\sqrt{2}$$

Assuming the car is not in reverse at A, $u = +8\sqrt{2}$

$$v = u + at$$
$$20 = 8\sqrt{2} + 0.34t$$
$$t = \frac{20 - 8\sqrt{2}}{0.34} = 25.5 \text{ (to 3 s.f.)}$$

The time taken for the car to travel from A to C is 25.5 s.

14 a For P, $a = 2$, $u = 4$

$$s = ut + \tfrac{1}{2}at^2$$
$$= 4t + \tfrac{1}{2} \times 2t^2 = 4t + t^2$$

The displacement of P is $(4t + t^2)$ m.

For Q, $a = 3.6$, $u = 3$

Q has been moving for $(t-1)$ seconds since passing through A, so

$$s = u(t-1) + \tfrac{1}{2}a(t-1)^2$$
$$= 3(t-1) + 1.8(t-1)^2 = 1.8t^2 - 0.6t - 1.2$$

The displacement of Q is $(1.8t^2 - 0.6t - 1.2)$ m.

b P and Q meet when $s_P = s_Q$, so, from **a**:

$$4t + t^2 = 1.8t^2 - 0.6t - 1.2$$
$$0.8t^2 - 4.6t - 1.2 = 0$$

Divide throughout by 0.2:

$$4t^2 - 23t - 6 = 0$$
$$(t-6)(4t+1) = 0$$

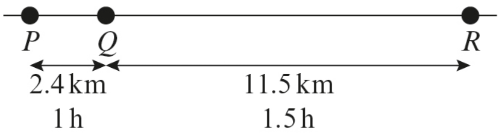
Rejecting a negative solution for time, $t = 6$.

c Substitute $t = 6$ into the equation for one of the displacements (here P):

$$s = 4t + t^2 = 4 \times 6 + 6^2 = 60$$

The distance of A from the point where the particles meet is 60 m.

15



a Let the velocity as the competitor passes point Q be v_Q

For PQ, $s = 2.4$, $t = 1$, $v = v_Q$

$$s = vt - \tfrac{1}{2}at^2$$
$$2.4 = v_Q \times 1 - \tfrac{1}{2}(a \times 1^2) = v_Q - \tfrac{1}{2}a$$
$$v_Q = 2.4 + 0.5a$$

For QR, $s = 11.5$, $t = 1.5$, $u = v_Q$

$$s = ut + \tfrac{1}{2}at^2$$
$$11.5 = v_Q \times 1.5 + \tfrac{1}{2}a \times 1.5^2 = 1.5v_Q + 1.125a$$

Substituting for v_Q:

$$11.5 = 1.5(2.4 + 0.5a) + 1.125a$$
$$= 3.6 + 0.75a + 1.125a$$
$$11.5 - 3.6 = (0.75 + 1.125)a$$
$$a = \frac{11.5 - 3.6}{0.75 + 1.125} = \frac{7.9}{1.875} = 4.21 \text{ (to 3 s.f.)}$$

The acceleration is 4.21 km h^{-2}.

$$4.21 \text{ km h}^{-2} = \frac{4.21 \times 1000}{3600 \times 3600} \text{ m s}^{-2} = 3.25 \times 10^{-4} \text{ m s}^{-2} \text{ (to 3 s.f.)}$$

So her acceleration is 3.25 x 10^{-4} m s^{-2}.

b For PQ, $s = 2.4$, $t = 1$, $a = 4.21$, $u = ?$, using exact figures

$$s = ut + \tfrac{1}{2}at^2$$
$$2.4 = u \times 1 + \tfrac{1}{2} \times \frac{7.9}{1.875} \times 1^2$$
$$u = 0.293 \text{ (to 3 s.f.)}$$

$$0.293 \text{ km h}^{-1} = \frac{0.293 \times 1000}{3600} \text{ m s}^{-1} = 0.0815 \text{ m s}^{-1} \text{ (to 3 s.f.)}$$

Constant acceleration 9E

1 a Take downwards as the positive direction.

$s = 28$, $u = 0$, $a = 9.8$, $t = ?$

$$s = ut + \tfrac{1}{2}at^2$$
$$28 = 0 \times t + \tfrac{1}{2} \times 9.8 \times t^2 = 4.9t^2$$
$$t = \sqrt{\frac{28}{4.9}} = 2.4 \text{ (to 2 s.f.)}$$

The time taken for the diver to hit the water is 2.4 s.

b $v^2 = u^2 + 2as$
$$v^2 = 0 + 2 \times 9.8 \times 28 = 548.8$$
$$v = \sqrt{548.8} = 32.4 \text{ (to 3 s.f.)}$$

When the diver hits the water, he is travelling at 32.4 m s^{-1}.

2 Take upwards as the positive direction.

$u = 20$, $a = -9.8$, $s = 0$, $t = ?$

$$s = ut + \tfrac{1}{2}at^2$$
$$0 = 20t - 4.9t^2 = t(20 - 4.9t), \quad t \neq 0$$
$$t = \tfrac{20}{4.9} = 4.1 \text{ (to 2 s.f.)}$$

The time of flight of the particle is 4.1 s.

3 Take downwards as the positive direction.

$u = 18$, $a = 9.8$, $t = 1.6$, $s = ?$

$$s = ut + \tfrac{1}{2}at^2 = 18 \times 1.6 + 4.9 \times 1.6^2 = 41 \text{ (to 2 s.f.)}$$

The height of the tower is 41 m.

4 a Take upwards as the positive direction.

$u = 24$, $a = -9.8$, $v = 0$, $s = ?$

$$v^2 = u^2 + 2as$$
$$0^2 = 24^2 - 2 \times 9.8 \times s$$
$$s = \frac{24^2}{2 \times 9.8} = 29 \text{ (to 2 s.f.)}$$

The greatest height reached by the pebble above the point of projection is 29 m.

4 b $u = 24,\ a = -9.8,\ v = 0,\ t = ?$

$v = u + at$
$0 = 24 - 9.8t$
$t = \frac{24}{9.8} = 2.4$ (to 2 s.f.)

The time taken to reach the greatest height is 2.4 s.

5 a Take upwards as the positive direction.

$u = 18,\ a = -9.8,\ s = 15,\ v = ?$

$v^2 = u^2 + 2as = 18^2 - 2 \times 9.8 \times 15 = 30$
$v = \sqrt{30} = \pm 5.5$ (to 2 s.f.)

The speed of the ball when it is 15 m above its point of projection is 5.5 m s^{-1}.

b $u = 18,\ a = -9.8,\ s = -4,\ v = ?$

$v^2 = u^2 + 2as = 18^2 + 2 \times (-9.8) \times (-4) = 324 + 78.4 = 402.4$
$v = -\sqrt{402.2} = -20$ (to 2 s.f.)

The speed with which the ball hits the ground is 20 m s^{-1}.

6 a Take downwards as the positive direction.

$s = 80,\ u = 4,\ a = 9.8,\ v = ?$

$v^2 = u^2 + 2as$
$= 4^2 + 2 \times 9.8 \times 80 = 1584$
$v = \sqrt{1584} = 40$ (to 2 s.f.)

The speed with which P hits the ground is 40 m s^{-1}.

b $u = 4,\ a = 9.8,\ v = \sqrt{1584},\ t = ?$

$v = u + at$
$\sqrt{1584} = 4 + 9.8t$
$$t = \frac{\sqrt{1584} - 4}{9.8} = 3.7 \text{ (to 2 s.f.)}$$

The time P takes to reach the ground is 3.7 s.

7 **a** Take upwards as the positive direction.

$v = -10, \ a = -9.8, \ t = 5, \ u = ?$

$$\begin{aligned} v &= u + at \\ -10 &= u - 9.8 \times 5 \\ u &= 9.8 \times 5 - 10 = 39 \end{aligned}$$

The speed of projection of P is 39 m s^{-1}.

b $u = 39, \ v = 0, \ a = -9.8, \ s = ?$

$$v^2 = u^2 + 2as$$
$$0^2 = 39^2 - 2 \times 9.8 \times s$$
$$s = \frac{1521}{2 \times 9.8} = 78 \text{ (to 2 s.f.)}$$

The greatest height above X attained by P during its motion is 78 m.

8 Take upwards as the positive direction.

$u = 21, \ t = 4.5, \ a = -9.8, \ s = ?$

$$s = ut + \tfrac{1}{2}at^2 = 21 \times 4.5 - 4.9 \times 4.5^2 = -4.7 \text{ (to 2 s.f.)}$$

The height above the ground from which the ball was thrown is 4.7 m.

9 Take upwards as the positive direction.

Find time when stone is instantaneously stationery:

$v = 0,\ u = 16,\ a = -9.8,\ t = ?$

$v = u + at$

$0 = 16 - 9.8t$

$t = \frac{16}{9.8} = 16.326... = 1.6$ s (to 1 d.p.)

So the stone is instantaneously stationary at 1.6 s

Find time of flight:

$s = -3,\ u = 16,\ a = -9.8,\ t = ?$

$s = ut + \frac{1}{2}at^2$

$-3 = 16t - 4.9t^2$

$4.9t^2 - 16t - 3 = 0$, so using the quadratic formula,

$$t = \frac{-(-16) \pm \sqrt{(-16)^2 - 4\times(4.9)\times(-3)}}{2\times(4.9)}$$

$t = 3.4431... = 3.4$ (to 1 d.p.) as we may discount the negative answer.

So the time of flight of the stone is 3.4 s.

Find speed when stone hits the ground:

$v = ?,\ u = 16,\ a = -9.8,\ t = 3.4431...$

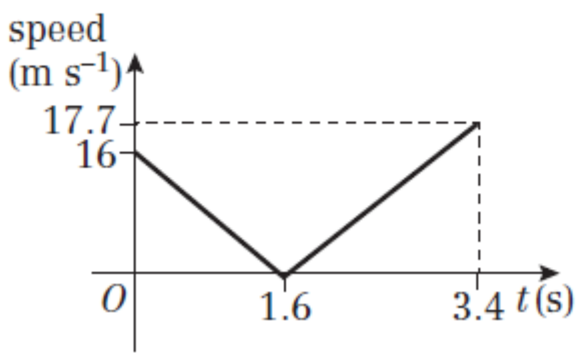
$v = u + at$

$v = 16 - 9.8\times 3.4431...$

$v = -17.74... = -17.7\ \text{ms}^{-1}$ (to 1 d.p.)

So speed when stone hits the ground is 17.7 ms^{-1}

Sketch speed-time graph



10 Take upwards as the positive direction.

$u = 24.5,\ a = -9.8,\ s = 21,\ t = ?$

$s = ut + \frac{1}{2}at^2$
$21 = 24.5t - 4.9t^2$

$4.9t^2 - 24.5t + 21 = 0$

Using the quadratic formula,

$$t = \frac{-(-24.5) \pm \sqrt{(-24.5)^2 - 4\times(4.9)\times(21)}}{2\times(4.9)}$$

$= 1.1 \text{ or } 3.9$

The difference between these times is
$(3.9 - 1.1)\ \text{s} = 2.8\ \text{s}$

The total time for which the particle is 21 m or more above its point of projection is 2.8 s.

11 a Take upwards as the positive direction.

$v = \frac{1}{3}u,\ a = -9.8,\ t = 2,\ u = ?$

$v = u + at$
$\frac{1}{3}u = u - 9.8\times 2$
$\frac{2}{3}u = 19.6 \Rightarrow u = \frac{3}{2}\times 19.6 = 29.4$

$u = 29$ (to 2 s.f.)

b $u = 29.4,\ s = 0,\ a = -9.8,\ t = ?$

$s = ut + \frac{1}{2}at^2$
$0 = 29.4t - 4.9t^2 = t(29.4 - 4.9t),\ t \neq 0$
$t = \frac{29.4}{4.9} = 6$

The time from the instant that the particle leaves O to the instant that it returns to O is 6 s.

12 For A, take downwards as the positive direction, $s_A = ut + \frac{1}{2}at^2 = 5t + 4.9t^2$

For B, take upwards as the positive direction, $s_B = ut + \frac{1}{2}at^2 = 18t - 4.9t^2$

$s_A + s_B = 46$

$(5t + 4.9t^2) + (18t - 4.9t^2) = 46$

$23t = 46 \Rightarrow t = 2$

Substitute $t = 2$ into $s_A = 5t + 4.9t^2$

$s_A = 5 \times 2 + 4.9 \times 2^2 = 29.6 = 30$ (to 2 s.f.)

The distance of the point where A and B collide from the point where A was thrown is 30 m.

13 a Find the speed, u_1 say, immediately before the ball strikes the floor.

$u = 0,\ a = 9.8,\ s = 10,\ v = u_1$
$v^2 = u^2 + 2as$
$u_1^2 = 0^2 + 2 \times 9.8 \times 10 = 196$
$u_1 = \sqrt{196} = 14$

The speed of the first rebound, u_2 say, is given by

$u_2 = \frac{3}{4}u_1 = \frac{3}{4} \times 14 = 10.5$

Find the maximum height, h_1 say, reached after the first rebound.

$u = 10.5,\ v = 0,\ a = -9.8,\ s = h_1$

$v^2 = u^2 + 2as$
$0^2 = 10.5^2 - 2 \times 9.8 \times h_1$

13 a $h_1 = \frac{10.5^2}{2 \times 9.8} = 5.6$ (to 2 s.f.)

The greatest height above the floor reached by the ball the first time it rebounds is 5.6 m.

b Immediately before the ball strikes the floor for the second time, its speed is again $u_2 = 10.5$ by symmetry. The speed of the second rebound, u_3 say, is given by

$u_3 = \frac{3}{4}u_2 = \frac{3}{4} \times 10.5 = 7.875$

Find the maximum height, h_2 say, reached after the second rebound.

$u = 7.875,\ v = 0,\ a = -9.8,\ s = h_2$

$$v^2 = u^2 + 2as$$
$$0^2 = 7.875^2 - 2 \times 9.8 \times h_2$$
$$h_2 = \frac{7.875^2}{2 \times 9.8} = 3.2 \text{ (to 2 s.f.)}$$

The greatest height above the floor reached by the ball the second time it rebounds is 3.2 m.

Challenge

1 a Take upwards as the positive direction.

For P, $s = ut + \frac{1}{2}at^2$ gives $s_P = 12t - 4.9t^2$

For Q, $s = ut + \frac{1}{2}at^2$

Q has been moving for 1 less second than P, so

$s_Q = 20(t-1) - 4.9(t-1)^2$

At the point of collision $s_P = s_Q$

$$12t - 4.9t^2 = 20(t-1) - 4.9(t-1)^2$$
$$= 20t - 20 - 4.9t^2 + 9.8t - 4.9$$

$24.9 = 17.8t \Rightarrow t = \frac{24.9}{17.8} = 1.4$ (to 2 s.f.)

The time between the instant when P is projected and the instant when P and Q collide is 1.4 s.

Challenge

1 b Substitute t into $s_P = 12t - 4.9t^2$ from part **a**

$s_P = 12t - 4.9t^2 \approx 12 \times 1.4 - 4.9 \times 1.4^2 = 7.2$ (to 2 s.f.)

The distance of the point where P and Q collide from O is 7.2 m.

2 Take downwards as positive.

For 1st stone: $u = 0$, $t = t_1$, $a = 9.8$, $s = h$

$s = ut + \frac{1}{2}at^2$

$h = 0 \times t_1 + \frac{1}{2} \times 9.8 \times t_1^2 = 4.9t_1^2$

For 2nd stone: $u = 25$, $t = t_1 - 2$, $a = 9.8$, $s = h$

$s = ut + \frac{1}{2}at^2$

$$\begin{aligned} h &= 25(t_1 - 2) + \tfrac{1}{2}(9.8 \times (t_1 - 2)^2) \\ &= 25t_1 - 50 + 4.9 \times (t_1^2 - 4t_1 + 4)) \\ &= 25t_1 - 50 + 4.9t_1^2 - 19.6t_1 + 19.6 \\ &= 4.9t_1^2 + 5.4t_1 - 30.4 \end{aligned}$$

Substituting for h from information for first stone:

$$\begin{aligned} 4.9t_1^2 &= 4.9t_1^2 + 5.4t_1 - 30.4 \\ 30.4 &= 5.4t_1 \\ t_1 &= \frac{30.4}{5.4} = 5.629 \end{aligned}$$

Putting this value into equation for first stone:

$h = 4.9 \times 5.629^2 = 4.9 \times 31.69 = 155$ (to 3 s.f.)

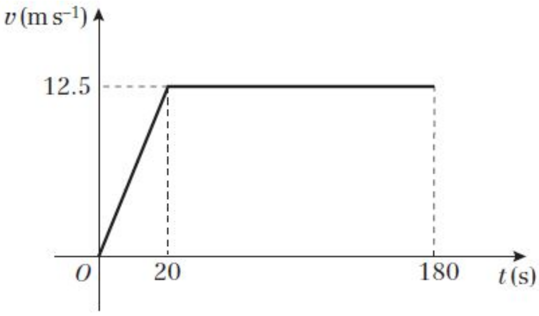
The height of the building is 155 m.

Constant acceleration, Mixed Exercise 9

1 a $45 \text{ km h}^{-1} = \frac{45 \times 1000}{3600} \text{ m s}^{-1}$

$= 12.5 \text{ m s}^{-1}$

$3 \text{ min} = 180 \text{ s}$

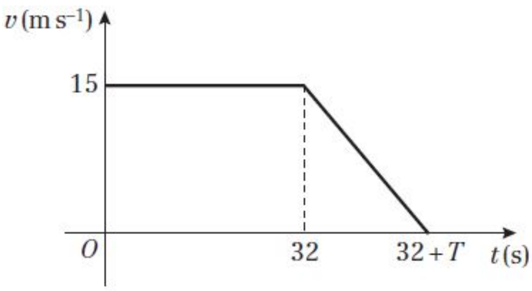


b $s = \frac{1}{2}(a+b)h$

$= \frac{1}{2}(160+180) \times 12.5 = 2125$

The distance from A to B is 2125 m.

2 a



b $s = \frac{1}{2}(a+b)h$

$570 = \frac{1}{2}(32+32+T) \times 15$

$\frac{15}{2}(T+64) = 570$

$T + 64 = \frac{570 \times 2}{15} = 76$

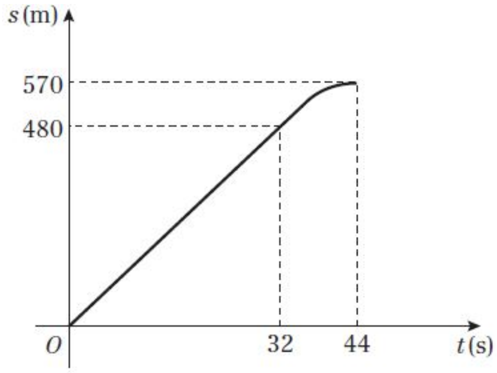
$T = 76 - 64 = 12$

c At $t = 32$, $s = 32 \times 15 = 480$

At $t = 44$, $s = 480 + \text{area of the triangle}$

$= 480 + \frac{1}{2} \times 12 \times 15 = 570$

2 c



3 a i Gradient of line = $\frac{v-u}{t}$

$a = \frac{v-u}{t}$

Rearranging: $v = u + at$

ii Shaded area is a trapezium

area = $\left(\frac{u+v}{2}\right)t$

$s = \left(\frac{u+v}{2}\right)t$

b i Rearrange $v = u + at$

$t = \frac{v-u}{a}$

Substitute into $s = \left(\frac{u+v}{2}\right)t$

$s = \left(\frac{u+v}{2}\right)\left(\frac{v-u}{a}\right)$

$2as = v^2 - u^2$
$v^2 = u^2 + 2as$

ii Substitute $v = u + at$ into $s = \left(\frac{u+v}{2}\right)t$

$s = \left(\frac{u+u+at}{2}\right)t$

$s = \left(\frac{2u}{2} + \frac{at}{2}\right)t$

$s = ut + \frac{1}{2}at^2$

3 b iii Substitute $u = v - at$ into $s = ut + \frac{1}{2}at^2$

$s = (v - at)t + \frac{1}{2}at^2$

$s = vt - \frac{1}{2}at^2$

4 $s = \frac{1}{2}(a + b)h$

$152 = \frac{1}{2}(15 + 23)u = 19u$

$u = \frac{152}{19} = 8$

5 $40\text{ km h}^{-1} = \frac{40 \times 1000}{3600}\text{ m s}^{-1} = \frac{100}{9}\text{ m s}^{-1}$

$24\text{ km h}^{-1} = \frac{24 \times 1000}{3600}\text{ m s}^{-1} = \frac{20}{3}\text{ m s}^{-1}$

$u = \frac{100}{9},\ v = \frac{20}{3},\ s = 240,\ a = ?$

$v^2 = u^2 + 2as$

$\left(\frac{20}{3}\right)^2 = \left(\frac{100}{9}\right)^2 + 2 \times a \times 240$

$a = \frac{\left(\frac{20}{3}\right)^2 - \left(\frac{100}{9}\right)^2}{2 \times 240} = -0.165$ (to 2 s.f.)

The deceleration of the car is 0.165 m s^{-2}.

6 a $a = -2.5,\ u = 20,\ t = 12,\ s = ?$

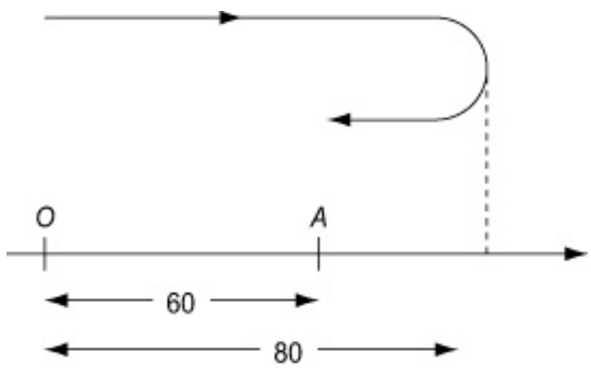
$s = ut + \frac{1}{2}at^2$

$= 20 \times 12 - \frac{1}{2} \times 2.5 \times 12^2$

$= 240 - 180 = 60$

$OA = 60$ m

b The particle will turn round when $v = 0$



$a = -2.5,\ u = 20,\ v = 0,\ s = ?$

$v^2 = u^2 + 2as$

$0^2 = 20^2 - 5s \Rightarrow s = 80$

The total distance P travels is $(80 + 20)$ m $= 100$ m

7 $u = 6,\ v = 25,\ a = 9.8,\ t = ?$

$$v = u + at$$
$$25 = 6 + 9.8t$$
$$t = \frac{25-6}{9.8} = 1.9 \text{ (to 2 s.f.)}$$

The ball takes 1.9 s to move from the top of the tower to the ground.

8 Take downwards as the positive direction.

a $u = 0,\ s = 82,\ a = 9.8,\ t = ?$

$$s = ut + \tfrac{1}{2}at^2$$
$$82 = 0 + 4.9t^2$$
$$t = \sqrt{\frac{82}{4.9}} = 4.1 \text{ (to 2 s.f.)}$$

The time taken for the ball to reach the sea is 4.1 s.

b $u = 0$, $s = 82$, $a = 9.8$, $v = ?$

$$v^2 = u^2 + 2as$$
$$= 0 + 2 \times 9.8 \times 82 = 1607.2$$
$$v = \sqrt{1607.2} = 40 \text{ (to 2 s.f.)}$$

The speed at which the ball hits the sea is 40 m s^{-1}.

c Air resistance/wind/turbulence

9 a distance = area of triangle + area of rectangle + area of trapezium

$$451 = \tfrac{1}{2} \times 8 \times 2u + 12 \times 2u + \tfrac{1}{2} \times (u + 2u) \times 6$$
$$= 8u + 24u + 9u = 41u$$
$$u = \tfrac{451}{41} = 11$$

b The particle is moving with speed less than u m s^{-1} for the first 4 s

$$s = \tfrac{1}{2} \times 4 \times 11 = 22$$

The distance moved with speed less than u m s^{-1} is 22 m.

10 a From O to P, $u = 18,\ t = 12,\ v = 24,\ a = ?$

$u = 18,\ t = 12,\ v = 24,\ a = ?$

$$v = u + at$$
$$24 = 18 + 12a$$
$$a = \frac{24 - 18}{12} = \frac{1}{2}$$

From O to Q, $u = 18,\ t = 20,\ a = \frac{1}{2},\ v = ?$

$$v = u + at$$
$$= 18 + \tfrac{1}{2} \times 20 = 28$$

The speed of the train at Q is 28 m s^{-1}.

b From P to Q

$u = 24,\ v = 28,\ t = 8,\ s = ?$

$$s = \left(\frac{u + v}{2}\right)t = \left(\frac{24 + 28}{2}\right) \times 8 = 208$$

The distance from P to Q is 208 m.

11 a $s = 104,\ t = 8,\ v = 18,\ u = ?$

$$s = \left(\frac{u + v}{2}\right)t$$
$$104 = \left(\frac{u + 18}{2}\right) \times 8 = (u + 18) \times 4 = 4u + 72$$
$$u = \frac{104 - 72}{4} = 8$$

The speed of the particle at X is 8 m s^{-1}

b $s = 104,\ t = 8,\ v = 18,\ a = ?$

$$s = vt - \tfrac{1}{2}at^2$$
$$104 = 18 \times 8 - \tfrac{1}{2}a \times 8^2 = 144 - 32a$$
$$a = \frac{144 - 104}{32} = 1.25$$

The acceleration of the particle is 1.25 m s^{-2}.

11 c From X to Z, $u = 8,\ v = 24,\ a = 1.25,\ s = ?$

$$v^2 = u^2 + 2as$$
$$24^2 = 8^2 + 2 \times 1.25 \times s$$
$$s = \frac{24^2 - 8^2}{2.5} = 204.8$$

$XZ = 204.8$ m

12 a Take upwards as the positive direction.

$u = 21,\ s = -32,\ a = -9.8,\ v = ?$

$$v^2 = u^2 + 2as$$
$$= 21^2 + 2 \times (-9.8) \times (-32) = 441 + 627.2 = 1068.2$$
$$v = \sqrt{1068.2} = \pm 33 \text{ (to 2 s.f.)}$$

The velocity with which the pebble strikes the ground is -33 m s^{-1}.
The speed is 33 m s^{-1}.

b 40 m above the ground is 8 m above the point of projection.

$u = 21,\ s = 8,\ a = -9.8,\ t = ?$
$s = ut + \frac{1}{2}at^2$
$8 = 21t - 4.9t^2$

$0 = 4.9t^2 - 21t + 8$, so using the quadratic formula,

$$t = \frac{21 \pm \sqrt{21^2 - 4 \times 4.9 \times 8}}{9.8} = \frac{21 \pm \sqrt{284.2}}{9.8} = 3.86,\ 0.423 \text{ (to 3 s.f.)}$$

The pebble is above 40 m between these times: $3.863... - 0.423... = 3.4$ (to 2 s.f.)
The pebble is more than 40 m above the ground for 3.4 s.

12 c Take upwards as the positive direction.

$u = 21,\ a = -9.8$

$v = u + at = 21 - 9.8t \Rightarrow t = \dfrac{21 - v}{9.8}$

From part **a**, the pebble hits the ground when $v = -33$.

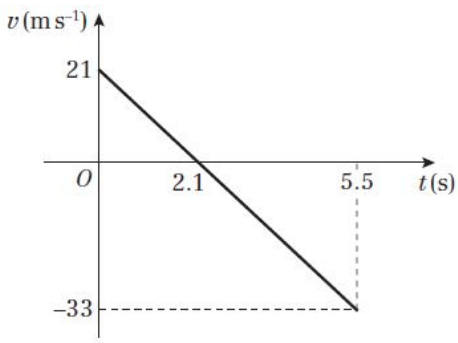
$t = \dfrac{21 - v}{9.8} = \dfrac{21 - (-33)}{9.8} = \dfrac{54}{9.8} = 5.5$ (to 2 s.f.)

This is shown on the graph at point $(5.5, -33)$

The graph crosses the t-axis when $v = 0$.

$t = \dfrac{21 - v}{9.8} = \dfrac{21 - 0}{9.8} = \dfrac{21}{9.8} = 2.1$ (to 2 s.f.)

So the graph passes through point $(2.1,\ 0)$



13 a $u = 12,\ v = 32,\ s = 1100,\ t = ?$

$s = \left(\dfrac{u + v}{2}\right)t$

$1100 = \left(\dfrac{12 + 32}{2}\right)t = 22t \Rightarrow t = \dfrac{1100}{22} = 50$

The time taken by the car to move from A to C is 50 s.

13 b Find a first.

From A to C, $u = 12,\ v = 32,\ t = 50,\ a = ?$

$$v = u + at$$
$$32 = 12 + a \times 50$$
$$a = \frac{32 - 12}{50} = 0.4$$

From A to B, $u = 12,\ s = 550,\ a = 0.4,\ v = ?$

$$v^2 = u^2 + 2as$$
$$= 12^2 + 2 \times 0.4 \times 550 = 584 \Rightarrow v = 24.2 \text{ (to 3 s.f.)}$$

The car passes B with speed 24.2 m s^{-1}.

14 Take upwards as the positive direction.

At the top:

$u = 30,\ v = 0,\ a = -9.8,\ t = ?$

$$v = u + at$$
$$0 = 30 - 9.8t \Rightarrow t = \frac{30}{9.8}$$

The ball spends 2.4 seconds above h, thus (by symmetry) 1.2 seconds rising between h and the top.
So it passes h 1.2 seconds earlier, at $t = \frac{30}{9.8} - 1.2 = 1.86$ (to 3 s.f.)

At h, $u = 30,\ a = -9.8,\ t \approx 1.86, s = ?$

$$s = ut + \tfrac{1}{2}at^2$$
$$= 30 \times 1.86 + \tfrac{1}{2}(-9.8) \times 1.86^2 = 39 \text{ (to 2 s.f.)}$$

15 a $u = 20,\ a = 4,\ s = 78,\ v = ?$

$$v^2 = u^2 + 2as$$
$$= 20^2 + 2 \times 4 \times 78 = 1024$$
$$v = \sqrt{1024} = 32$$

The speed of B when it has travelled 78 m is 32 m s^{-1}.

15 b Find time for B to reach the point 78 m from O.

$v = 32,\ u = 20,\ a = 4,\ t = ?$

$v = u + at$

$32 = 20 + 4t \Rightarrow t = \dfrac{32-20}{4} = 3$

For A, distance = speed × time

$s = 30 \times 3 = 90$

The distance from O of A when B is 78 m from O is 90 m.

c At time t seconds, for A, $s = 30t$

for B, $s = ut + \frac{1}{2}at^2 = 20t + 2t^2$

On overtaking the distances are the same.

$20t + 2t^2 = 30t$

$t^2 - 5t = t(t-5) = 0$

$t = 5$ (at $t = 0$, A overtakes B)

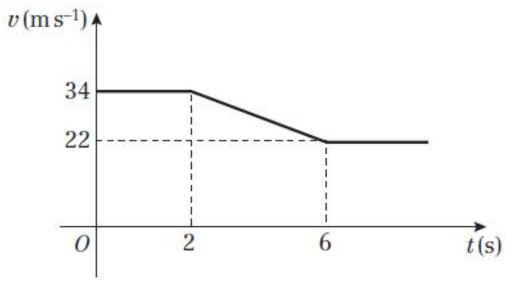
B overtakes A 5 s after passing O.

16 a To find time decelerating:

$u = 34,\ v = 22,\ a = -3,\ t = ?$

$v = u + at$

$22 = 34 - 3t \Rightarrow t = \dfrac{34-22}{3} = 4$

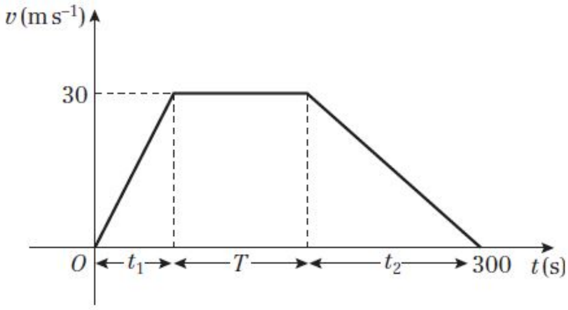


16 b distance = rectangle + trapezium

$$s = 34\times 2 + \tfrac{1}{2}(22+34)\times 4$$
$$= 68 + 112 = 180$$

Distance required is 180 m.

17 a



b Acceleration is the gradient of a line.

For the first part of the journey, $3x = \dfrac{30}{t_1} \Rightarrow t_1 = \dfrac{30}{3x} = \dfrac{10}{x}$

For the last part of the journey, $-x = -\dfrac{30}{t_2} \Rightarrow t_2 = \dfrac{30}{x}$

$$t_1 + T + t_2 = 300$$

$\frac{10}{x} + T + \frac{30}{x} = 300 \Rightarrow \frac{40}{x} + T = 300,$ as required

c $s = \tfrac{1}{2}(a+b)h$

$$6000 = \tfrac{1}{2}(T+300)\times 30 = 15T + 4500$$
$$T = \frac{6000-4500}{15} = 100$$

Substitute into the result in part **b**:

$$\tfrac{40}{x} + 100 = 300 \Rightarrow \tfrac{40}{x} = 200$$
$$x = \tfrac{40}{200} = 0.2$$

d From part **c**, $T = 100$

At constant velocity, distance = velocity × time $= 30\times 100 = 3000$ (m)

The distance travelled at a constant speed is 3 km.

17 e From part **b**, $t_1 = \frac{10}{x} = \frac{10}{0.2} = 50$

Total distance travelled = 6 km (given) so halfway = 3 km = 3000 m

While accelerating, distance travelled is $\left(\frac{1}{2}\times 50\times 30\right)$ m = 750 m.

At constant velocity, the train must travel a further 2250 m.

At constant velocity, time $= \dfrac{\text{distance}}{\text{velocity}} = \dfrac{2250}{30}$ s = 75 s

Time for train to reach halfway is $(50+75)\text{ s} = 125\text{ s}$

Challenge

Find the time taken by the first ball to reach 25 m below its point of projection (25 m above the ground). Take upwards as the positive direction.

$u = 10,\ s = -25,\ a = -9.8,\ t = ?$

$$\begin{aligned}
s &= ut + \tfrac{1}{2}at^2 \\
-25 &= 10t - 4.9t^2 \\
0 &= 4.9t^2 - 10t - 25 \\
t &= 10 \pm \frac{\sqrt{102 + 4\times 4.9\times 25}}{9.8} \\
&= 3.5 \text{ (to 2 s.f.)}
\end{aligned}$$

As we discard the negative solution. Find the time taken by the second ball to reach 25 m below its point of projection. Take downwards as the positive direction.

$u = 0,\ s = 25,\ a = 9.8,\ t = ?$

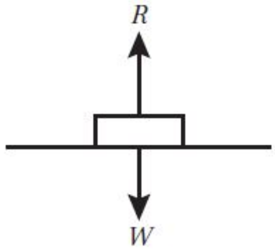
$$\begin{aligned}
s &= ut + \tfrac{1}{2}at^2 \\
25 &= 4.9t^2 \\
t &= 2.3 \text{ (to 2 s.f.)}
\end{aligned}$$

Combining the two results:

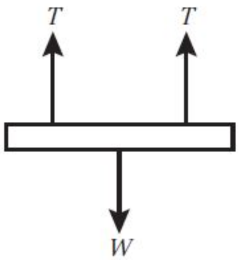
$T = 3.4989\ldots - 2.2587\ldots = 1.2$ (to 2 s.f. using exact figures)

Forces and motion 10A

1 R is the normal reaction of the table on the box.
W is the weight of the box.

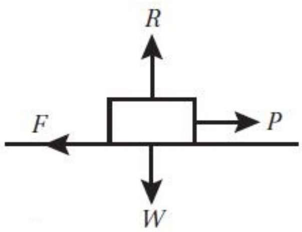


2 T is the tension in each of the ropes.
W is the weight of the bar.



3 W is the weight of the apple.

4 R is the normal reaction of the road on the car.
W is the weight of the car.
F is the sum of the frictional forces on the car.
P is the forward force produced by the car's engine.



5 W is the weight of the rescuer.
T is the tension in the rope.

6 Although its speed is constant, the satellite is continuously changing direction. This means the velocity changes. Therefore, there must be a resultant force acting on the satellite.

7 5 N

8 Since each particle is stationary, the overall force in each case is zero.

a Considering vertical forces:
$P - 10 = 0$
$P = 10$ N

8 b Considering horizontal forces only:
$P - 30 = 0$
$P = 30$ N

c Considering horizontal forces only:
$P + 1.5P - 50 = 0$
$2.5P = 50$
$P = 20$ N

9 a Since the platform is moving at constant velocity, the total vertical force is zero.
$T + T = 400$
$T = 200$
The tension in each rope is 200 N.

b If the tension in each rope is reduced by 50 N, there is a resultant downward force on the platform. It will therefore accelerate downward.

10 Since the particle is at rest, both horizontal and vertical forces must be balanced.
Considering horizontal forces only:
$p - 50 = 0$
$p = 50$
Considering vertical forces only:
$5q - (q + 10) - 3p = 0$
$4q - 10 - (3 \times 50) = 0$
$4q = 160$
$q = 40$
The values of p and q are 50 and 40 respectively.

11 Since the particle is moving with constant velocity, both horizontal and vertical forces must be balanced.
Considering horizontal forces only:
$2P + Q = 25$
$Q = 25 - 2P$
Considering vertical forces only:
$3P - 2Q = 20$
Substituting for Q:
$3P - 2 \times (25 - 2P) = 20$
$3P - 50 + 4P = 20$
$7P = 20 + 50 = 70$
$P = 10$ N
Using this value of P in the horizontal equation:
$(2 \times 10) + Q = 25$
$Q = 25 - 20 = 5$
$Q = 5$ N
P is 10 N and Q is 5 N.

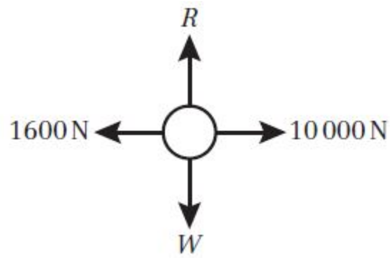
12 a i Overall horizontal force = 100 – 100 = 0
Overall vertical force = 40 – 20 = 20
The resultant force is 20 N upward.

ii The particle accelerates vertically upward.

12 b i Overall horizontal force = 25 – 5 = 20
Overall vertical force = 10 – 10 = 0
The resultant force is 20 N to the right.

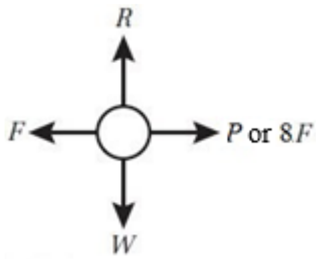
ii The particle accelerates to the right.

13 a R is the normal reaction of the road on the car.
W is the weight of the car.
The forward thrust of the car's engine acts to the right in the diagram.
The car is travelling to the right (positive direction).
The frictional forces on the car are acting to the left.



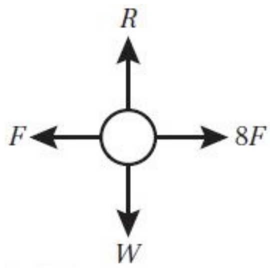
b Considering horizontal forces only:
Resultant force = 10 000 – 1600
There is no overall vertical force: R and W must be balanced, otherwise the car would lift off the road or sink into it.
The resultant force is 8400 N in the direction of travel.

14 a R is the normal reaction of the road on the car.
W is the weight of the car.
P is the driving force produced by the car's engine.
F is the resistance to the car's motion. or



b The magnitude of the driving force is eight times the magnitude of the resistance force, so

$P = 8F$



The resultant force is the difference between the forward force P and the resistance force F, so

$8F - F = 7F = 4200$

$F = \frac{4200}{7} = 600$

The magnitude of the resistance force is 600 N.

Forces and motion 10B

1 a $(-\mathbf{i} + 3\mathbf{j}) + (4\mathbf{i} - \mathbf{j}) = (3\mathbf{i} + 2\mathbf{j})$
The resultant force is $(3\mathbf{i} + 2\mathbf{j})$ N.

b $\begin{pmatrix}5\\3\end{pmatrix}+\begin{pmatrix}-3\\-6\end{pmatrix}=\begin{pmatrix}2\\-3\end{pmatrix}$

The resultant force is $\begin{pmatrix}2\\-3\end{pmatrix}$N.

c $(\mathbf{i} + \mathbf{j}) + (5\mathbf{i} - 3\mathbf{j}) + (-2\mathbf{i} - \mathbf{j}) = (4\mathbf{i} - 3\mathbf{j})$
The resultant force is $(4\mathbf{i} - 3\mathbf{j})$ N.

d $\begin{pmatrix}-1\\4\end{pmatrix}+\begin{pmatrix}6\\0\end{pmatrix}+\begin{pmatrix}-2\\-7\end{pmatrix}=\begin{pmatrix}3\\-3\end{pmatrix}$

The resultant force is $\begin{pmatrix}3\\-3\end{pmatrix}$N.

2 a $\mathbf{F}_1+\mathbf{F}_2+\mathbf{F}_3=0$

$\Rightarrow (2\mathbf{i}+7\mathbf{j})+(-3\mathbf{i}+\mathbf{j})+\mathbf{F}_3=0$

$\Rightarrow \mathbf{F}_3=-(2\mathbf{i}+7\mathbf{j})-(-3\mathbf{i}+\mathbf{j})$

$=-2\mathbf{i}-7\mathbf{j}+3\mathbf{i}-\mathbf{j}$

$=\mathbf{i}-8\mathbf{j}$

b $\mathbf{F}_1+\mathbf{F}_2+\mathbf{F}_3=0$

$\Rightarrow (3\mathbf{i}-4\mathbf{j})+(2\mathbf{i}+3\mathbf{j})+\mathbf{F}_3=0$

$\Rightarrow \mathbf{F}_3=-(3\mathbf{i}-4\mathbf{j})-(2\mathbf{i}+3\mathbf{j})$

$=-3\mathbf{i}+4\mathbf{j}-2\mathbf{i}-3\mathbf{j}$

$=-5\mathbf{i}+\mathbf{j}$

3 Since object is in equilibrium:

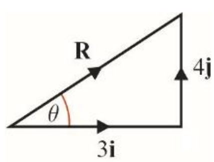
$\begin{pmatrix}a\\2b\end{pmatrix}+\begin{pmatrix}-2a\\-b\end{pmatrix}+\begin{pmatrix}3\\-4\end{pmatrix}=\begin{pmatrix}0\\0\end{pmatrix}$

$\begin{pmatrix}a\\2b\end{pmatrix}+\begin{pmatrix}-2a\\-b\end{pmatrix}=\begin{pmatrix}-3\\4\end{pmatrix}$

$\begin{pmatrix}-a\\b\end{pmatrix}=\begin{pmatrix}-3\\4\end{pmatrix}$

$a = 3$ and $b = 4$

4 a $(3\mathbf{i} + 4\mathbf{j})$



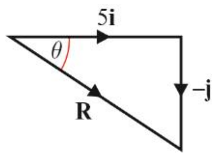
4 a i $\sqrt{3^2+4^2}=\sqrt{25}$

The resultant force is 5 N.

ii $\tan\theta=\frac{4}{3}$

The force makes an angle of 53.1° with **i**.

b (5**i** – **j**)



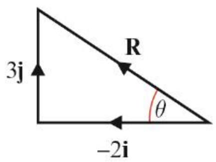
i $\sqrt{5^2+1^2}=\sqrt{26}$

The resultant force is $\sqrt{26}$ N.

ii $\tan\theta=\frac{1}{5}$

The force makes an angle of 11.3° with **i**.

c (−2**i** + 3**j**)



i $\sqrt{2^2+3^2}=\sqrt{13}$

The resultant force is $\sqrt{13}$ N.

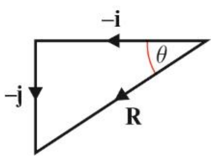
ii $\tan\theta=\frac{3}{2}$

$\theta=56.3°$ This is the angle made with the negative **i** vector

Angle made with the positive **i** vector = 180 − θ

The force makes an angle of 123.7° with **i**.

d



i $\sqrt{1^2+1^2}=\sqrt{2}$

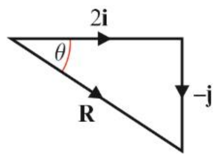
The resultant force is $\sqrt{2}$ N.

4 d ii $\tan\theta = \frac{1}{1}$

$\theta = 45°$. This is the angle made with the negative **i** vector.
The obtuse angle made with the positive **i** vector = $180 - \theta$
The force makes an angle of 135° with **i**.

5 a i $(-2\mathbf{i} + \mathbf{j}) + (5\mathbf{i} + 2\mathbf{j}) + (-\mathbf{i} - 4\mathbf{j}) = (2\mathbf{i} - \mathbf{j})$
The resultant vector is $(2\mathbf{i} - \mathbf{j})$ N.

ii $\sqrt{2^2 + 1^2} = \sqrt{5}$
The magnitude of the resultant vector is $\sqrt{5}$ N.

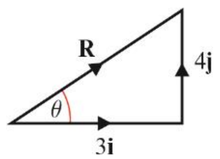


iii $\tan\theta = \frac{1}{2}$

$\theta = -26.6°$ This is the angle made from **east**, with **anticlockwise** defined as positive.
The **bearing** is the angle made from **north**, with **clockwise** defined as positive = $90 - \theta$
The force acts at a bearing of 116.6°.

b i $(-2\mathbf{i} + \mathbf{j}) + (2\mathbf{i} - 3\mathbf{j}) + (3\mathbf{i} + 6\mathbf{j}) = (3\mathbf{i} + 4\mathbf{j})$
The resultant vector is $(3\mathbf{i} + 4\mathbf{j})$ N

ii $\sqrt{3^2 + 4^2} = \sqrt{25}$
The resultant force is 5 N.



iii $\tan\theta = \frac{4}{3}$

$\theta = 53.1°$ This is the angle made from **east**, with **anticlockwise** defined as positive.
The **bearing** is the angle made from **north**, with **clockwise** defined as positive = $90 - \theta$
The force acts at a bearing of 036.9°.

6 Since the object is in equilibrium:
$(a\mathbf{i} - b\mathbf{j}) + (b\mathbf{i} + a\mathbf{j}) + (-4\mathbf{i} - 2\mathbf{j}) = 0$
Considering **i** components:
$a + b - 4 = 0$
so $b = 4 - a$ **(1)**
Considering **j** components:
$-b + a - 2 = 0$
Substituting $b = 4 - a$ from **(1)**:
$-(4 - a) + a - 2 = 0$
$2a = 2 + 4 = 6$
$a = 3$ **(2)**

6 Substituting **(2)** into **(1)**:

$b = 4 - 3 = 1$
The values of a and b are 3 and 1, respectively.

7 Since the object is in equilibrium:
$(2a\mathbf{i} + 2b\mathbf{j}) + (-5b\mathbf{i} + 3a\mathbf{j}) + (-11\mathbf{i} - 7\mathbf{j}) = 0$

Considering **i** components:
$2a - 5b - 11 = 0$ **(1)**

Considering **j** components:
$2b + 3a - 7 = 0$ **(2)**

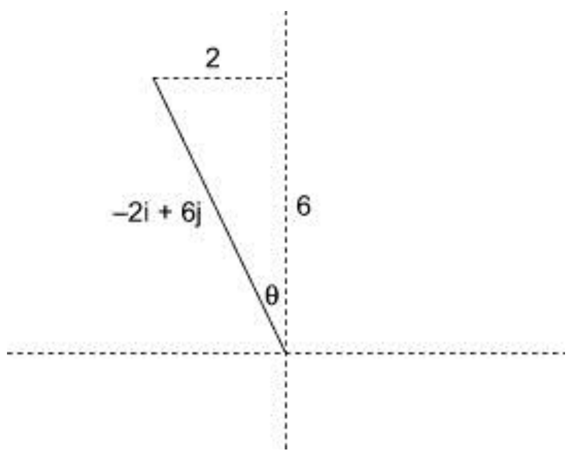
equation **(1)** × 3 → $6a - 15b - 33 = 0$ **(3)**
equation **(2)** × 2 → $6a + 4b - 14 = 0$ **(4)**

Subtracting (**4**) from (**3**):
$-15b - 33 - 4b - (-14) = 0$
$-19b = 33 - 14$
$b = -1$
Substituting this value into equation (**1**):
$2a - 5(-1) - 11 = 0$
$2a = 11 - 5 = 6$
The values of a and b are 3 and −1, respectively.

8 a $\mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3 = 0$
$\Rightarrow (-3\mathbf{i} + 7\mathbf{j}) + (\mathbf{i} - \mathbf{j}) + (p\mathbf{i} + q\mathbf{j}) = 0$
$(-3 + 1 + p)\mathbf{i} + (7 - 1 + q)\mathbf{j} = 0$
$p = 2,\ q = -6$

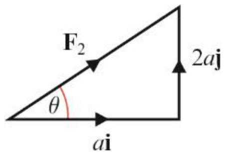
b $\mathbf{R} = \mathbf{F}_1 + \mathbf{F}_2$
$= (-3\mathbf{i} + 7\mathbf{j}) + (\mathbf{i} - \mathbf{j})$
$= -2\mathbf{i} + 6\mathbf{j}$
$|\mathbf{R}| = \sqrt{(-2)^2 + 6^2}$
$= \sqrt{4 + 36}$
$= \sqrt{40}$
$= 6.32$ N

c



$\tan\theta = \frac{2}{6}$
$\theta = 18°$

9 a $\mathbf{F}_2 = (a\mathbf{i} + 2a\mathbf{j})$



$\tan\theta = \frac{2a}{a} = 2$

$\mathbf{F}_2$ makes an angle of 63.4° with **i**.

b $\mathbf{R} = \mathbf{F}_1 + \mathbf{F}_2 = (3\mathbf{i} - 2\mathbf{j}) + (a\mathbf{i} + 2a\mathbf{j})$

i vector = $3 + a$ **j** vector = $-2 + 2a$

In the vector $(13\mathbf{i} + 10\mathbf{j})$:
i vector = 13 **j** vector = 10

Let θ_1 = the angle of vector **R** and θ_2 = the angle of vector $(13\mathbf{i} + 10\mathbf{j})$

Since the vectors are parallel, $\theta_1 = \theta_2$ so $\tan\theta_1 = \tan\theta_2$:

$\tan\theta_1 = \frac{\mathbf{j}\text{ vector}}{\mathbf{i}\text{ vector}} = \frac{-2+2a}{3+a}$ $\tan\theta_2 = \frac{\mathbf{j}\text{ vector}}{\mathbf{i}\text{ vector}} = \frac{10}{13}$

$\Rightarrow \frac{-2+2a}{3+a} = \frac{10}{13}$

$(-2 + 2a) \times 13 = (3 + a) \times 10$

$16a = 56$

$a = 3.5$

10 a Since the particle P is in equilibrium:

$\mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3 = 0$

$\begin{pmatrix}-7\\-4\end{pmatrix} + \begin{pmatrix}4\\2\end{pmatrix} + \begin{pmatrix}a\\b\end{pmatrix} = \begin{pmatrix}0\\0\end{pmatrix}$

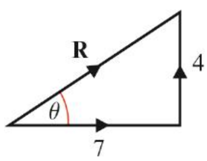
$\begin{pmatrix}a\\b\end{pmatrix} = \begin{pmatrix}7\\4\end{pmatrix} + \begin{pmatrix}-4\\-2\end{pmatrix}$

$\begin{pmatrix}a\\b\end{pmatrix} = \begin{pmatrix}3\\2\end{pmatrix}$

The values are $a = 3$, $b = 2$

b $\mathbf{R} = \mathbf{F}_2 + \mathbf{F}_3$

$\mathbf{R} = \begin{pmatrix}4\\2\end{pmatrix} + \begin{pmatrix}3\\2\end{pmatrix} = \begin{pmatrix}7\\4\end{pmatrix}$



10 b i $|\mathbf{R}| = \sqrt{7^2 + 4^2} = \sqrt{65}$

The magnitude of **R** is $\sqrt{65}$ N.

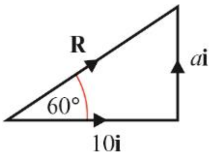
ii $\tan\theta = \frac{4}{7}$

$\theta = 29.7\ldots°$

R acts at 30° above the horizontal (to 2 s.f.)

Challenge

Redrawing the diagram as a closed triangle:



$\tan 60 = \frac{a}{10}$

$a = 10\tan 60 = 10 \times \sqrt{3}$

$\mathbf{R} = \begin{pmatrix} 10 \\ a \end{pmatrix} = \begin{pmatrix} 10 \\ 10\sqrt{3} \end{pmatrix}$

$|\mathbf{R}| = \sqrt{10^2 + \left(10\sqrt{3}\right)^2} = \sqrt{100 + 300} = \sqrt{400}$

The value of a is 17.3 (to 3 s.f.), and the magnitude of the resultant force is 20 N.

Forces and motion 10C

1 $F = ma$

$120 = 400a$

$a = 0.3$

The acceleration is $0.3\,\text{m}\,\text{s}^{-2}$

2 $W = mg$

$= 4 \times 9.8$

$= 39.2$

The weight of the particle is 39.2 N

3 $F = ma$

$30 = 1.2m$

$m = 25$

The mass of the object is 25 kg .

4 On Earth: $W = 735$ N, $g = 9.8$ m s^{-2}, m = ?

$W = mg$
$735 = m \times 9.8$
$m = 735 \div 9.8 = 75$ kg

On the moon: $W = 120$ N, g = ?, $m = 75$
$W = mg$
$120 = 75 \times g$
$g = 120 \div 75 = 1.6$

On the Moon, the acceleration due to gravity is 1.6 m s^{-2}.

5 Always resolve in the direction of acceleration.

a $R(\uparrow)$, $P - 2g = 2 \times 3$

$P = 25.6$

The magnitude of P is 25.6 N

b $R(\downarrow)$, $4g + 10 - P = 4 \times 2$

$49.2 - P = 8$

$P = 41.2$

The magnitude of P is 41.2 N

6 a $R(\downarrow), \quad mg - 10 = m \times 5$

$9.8m - 10 = 5m$

$m = 2.1 \quad (2\text{ s.f.})$

The mass of the body is 2.1 kg

b $R(\uparrow), \quad 20 - mg = m \times 2$

$20 - 9.8m = 2m$

$m = 1.7 \quad (2\text{ s.f.})$

The mass of the body is 1.7 kg

7 a $R(\downarrow), \quad 2g - 8 = 2a$

$5.8 = a$

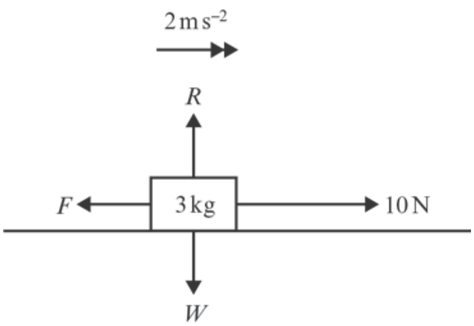
The acceleration of the body is $5.8\,\text{m s}^{-2}$

b $R(\uparrow), \quad 100 - 8g = 8a$

$2.7 = a$

The acceleration of the body is $2.7\,\text{m s}^{-2}$

8 W and R can be ignored, as they act at right angles to the motion.



Resultant force = ma
$m = 3$ kg, $a = 2$ ms^{-2}
$R(\rightarrow)$, $10 - F = 3 \times 2 = 6$
$F = 10 - 6$
The force due to friction is 4 N.

9 a $u = 0, \; v = 3, \; s = 5, \; a = ?$

$v^2 = u^2 + 2as$

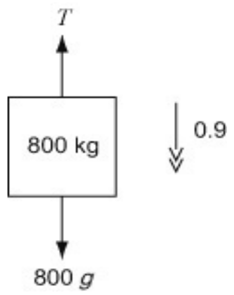
$3^2 = 0^2 + 2a \times 5$

$9 = 10a$

$a = 0.9$

The acceleration of the lift is $0.9\,\text{m s}^{-2}$

9 b



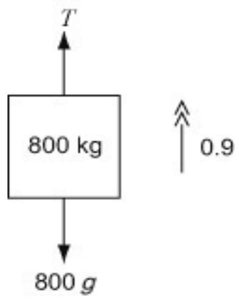
$R(\downarrow)$, $\quad 800g - T = 800 \times 0.9$

$7840 - T = 720$

$T = 7120$

The tension in the cable is 7120 N.

c



$R(\uparrow)$, $\quad T - 800g = 800 \times 0.9$

$T - 7840 = 720$

$T = 8560$

The tension in the cable is 8560 N.

10 a $u = 0,\ v = 1,\ t = 2,\ a = ?$

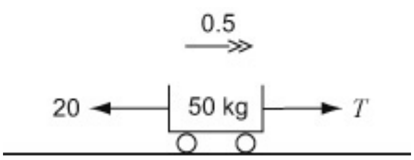
$v = u + at$

$1 = 0 + a \times 2$

$a = 0.5$

The acceleration of the trolley is $0.5\,\text{m}\,\text{s}^{-2}$

b

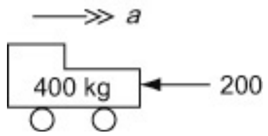


$R(\rightarrow)$, $\quad T - 20 = 50 \times 0.5$

$T = 45$

The tension in the rope is 45 N.

11 a



$R(\rightarrow)$, $-200 = 400a$
$a = -0.5$

$u = 16,\ v = 0,\ a = -0.5,\ t = ?$
$v = u + at \quad (\rightarrow)$
$0 = 16 - 0.5t$
$0.5t = 16$
$t = 32$

It takes 32 s for the van to stop.

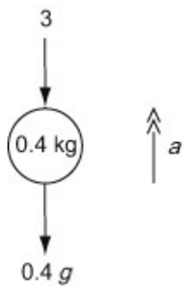
b $u = 16,\ v = 0,\ a = -0.5,\ s = ?$
$v^2 = u^2 + 2as \quad (\rightarrow)$
$0^2 = 16^2 + 2(-0.5)s$
$0 = 256 - s$
$s = 256$

The van travels 256 m before it stops.

c Air resistance is unlikely to be of constant magnitude. (It is usually a function of speed.)

Challenge

a

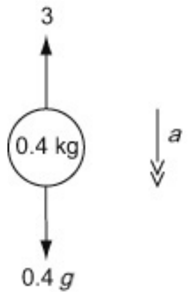


$R(\uparrow)$, $-3 - 0.4g = 0.4a$
$a = -17.3$

$u = 10,\ v = 0,\ a = -17.3,\ s = ?$
$v^2 = u^2 + 2as \quad (\uparrow)$
$0 = 10^2 + 2(-17.3)s$
$0 = 100 - 34.6s$
$s = 2.89\ldots = 2.9 \quad (2\text{ s.f.})$

The stone rises to a height of 2.9 m above the bottom of the pond.

b



$R(\downarrow)$, $\quad 0.4g - 3 = 0.4a$

$$0.92 = 0.4a$$

$$a = 2.3$$

$u = 0,\ s = \frac{100}{34.6},\ a = 2.3,\ v = ?$

$v^2 = u^2 + 2as \quad (\downarrow)$

$v^2 = 0^2 + 2 \times 2.3 \times \frac{100}{34.6}$

$v = 3.646.. = 3.6 \quad (2 \text{ s.f.})$

The stone hits the bottom of the pond with speed $3.6\,\text{m s}^{-1}$

c $u = 10,\ v = 0,\ a = -17.3,\ t = ?$

$v = u + at \quad (\uparrow)$

$0 = 10 - 17.3t,$

$t_1 = \frac{10}{17.3} = 0.57803\ldots$

$u = 0,\ a = 2.3,\ s = \frac{100}{34.6},\ t = ?$

$s = ut + \frac{1}{2}at^2 \quad (\downarrow)$

$\frac{100}{34.6} = 0 + \frac{1}{2} \times 2.3t_2^2$

$$t_2^2 = \frac{2 \times 100}{2.3 \times 34.6} = 2.51319$$

$t_2 = 1.585$

$t_1 + t_2 = 0.57803 + 1.585 = 2.16$

The total time is 2.16 s (3 s.f.)

Forces and motion 10D

1 a $F = (\mathbf{i} + 4\mathbf{j})$, $m = 2$, $\mathbf{a} = ?$

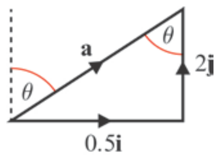
$F = m\mathbf{a}$

$(\mathbf{i} + 4\mathbf{j}) = 2\mathbf{a}$

$$\mathbf{a} = \frac{(\mathbf{i}+4\mathbf{j})}{2}$$

The acceleration of the particle is $(0.5\mathbf{i} + 2\mathbf{j})$ m s^{-2}.

b



$$|\mathbf{a}| = \sqrt{0.5^2 + 2^2} = \sqrt{4.25}$$

The magnitude of the acceleration is 2.06 m s^{-2}.

Using Z angles (see diagram), bearing = θ

$$\tan\theta = \frac{0.5}{2}$$

$\theta = 14°$

The bearing of the acceleration is 014°.

2 $F = (4\mathbf{i} + 3\mathbf{j})$, $\mathbf{a} = (20\mathbf{i} + 15\mathbf{j})$, $m = ?$

$F = m\mathbf{a}$

$(4\mathbf{i} + 3\mathbf{j}) = m \times (20\mathbf{i} + 15\mathbf{j})$

$$m = \frac{(4\mathbf{i}+3\mathbf{j})}{(20\mathbf{i}+15\mathbf{j})} = \frac{1}{5}$$

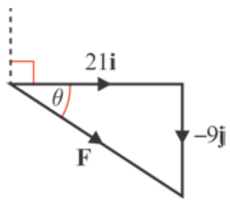
The mass of the particle is 0.2 kg.

3 a $\mathbf{a} = (7\mathbf{i} - 3\mathbf{j})$, $m = 3$, $F = $ **?**

$F = m\mathbf{a}$

$= 3 \times (7\mathbf{i} - 3\mathbf{j})$

$= (21\mathbf{i} - 9\mathbf{j})$



b

$$|\mathbf{F}| = \sqrt{21^2 + 9^2} = \sqrt{522}$$

The force has a magnitude of 22.8 N (3 s.f.)

$$\tan\theta = \frac{9}{21}$$

$\theta = 23.19\ldots°$

But bearing = $90° + \theta$ (see diagram)

The force acts at a bearing of 113° (to the nearest degree).

4 a $\mathbf{F}_1 = (2\mathbf{i} + 7\mathbf{j})$, $\mathbf{F}_2 = (-3\mathbf{i} + \mathbf{j})$, $m = 0.25$
$F = \mathbf{F}_1 + \mathbf{F}_2 = m\mathbf{a}$
$(2\mathbf{i} + 7\mathbf{j}) + (-3\mathbf{i} + \mathbf{j}) = 0.25\mathbf{a}$
$(-\mathbf{i} + 8\mathbf{j}) = 0.25\mathbf{a}$
$\mathbf{a} = \dfrac{(-\mathbf{i}+8\mathbf{j})}{0.25}$
The acceleration is $(-4\mathbf{i} + 32\mathbf{j})$ m s^{-2}.

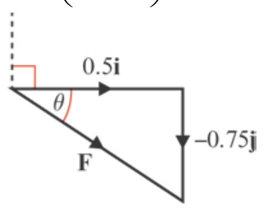
b $\mathbf{F}_1 = (3\mathbf{i} - 4\mathbf{j})$, $\mathbf{F}_2 = (2\mathbf{i} + 3\mathbf{j})$, $m = 6$
$F = \mathbf{F}_1 + \mathbf{F}_2 = m\mathbf{a}$
$(3\mathbf{i} - 4\mathbf{j}) + (2\mathbf{i} + 3\mathbf{j}) = 6\mathbf{a}$
$(5\mathbf{i} - \mathbf{j}) = 6\mathbf{a}$
$\mathbf{a} = \dfrac{(5\mathbf{i}-\mathbf{j})}{6}$
The acceleration is $\left(\dfrac{5}{6}\mathbf{i} - \dfrac{1}{6}\mathbf{j}\right)$ m s^{-2}.

c $\mathbf{F}_1 = (-40\mathbf{i} - 20\mathbf{j})$, $\mathbf{F}_2 = (25\mathbf{i} + 10\mathbf{j})$, $m = 15$
$F = \mathbf{F}_1 + \mathbf{F}_2 = m\mathbf{a}$
$(-40\mathbf{i} - 20\mathbf{j}) + (25\mathbf{i} + 10\mathbf{j}) = 15\mathbf{a}$
$(-15\mathbf{i} - 10\mathbf{j}) = 15\mathbf{a}$
$\mathbf{a} = \dfrac{(-15\mathbf{i}-10\mathbf{j})}{15}$
The acceleration is $\left(-\mathbf{i} - \dfrac{2}{3}\mathbf{j}\right)$ m s^{-2}.

d $\mathbf{F}_1 = 4\mathbf{j}$, $\mathbf{F}_2 = (-2\mathbf{i} + 5\mathbf{j})$, $m = 1.5$
$F = \mathbf{F}_1 + \mathbf{F}_2 = m\mathbf{a}$
$4\mathbf{j} + (-2\mathbf{i} + 5\mathbf{j}) = 1.5\mathbf{a}$
$(-2\mathbf{i} + 9\mathbf{j}) = 1.5\mathbf{a}$
$\mathbf{a} = \dfrac{(-2\mathbf{i}+9\mathbf{j})}{1.5}$
The acceleration is $\left(-\dfrac{4}{3}\mathbf{i} + 6\mathbf{j}\right)$ m s^{-2}.

5 a Resultant force, $F = \mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3$
$$F = \begin{pmatrix} 3 \\ -1 \end{pmatrix} + \begin{pmatrix} 2 \\ -5 \end{pmatrix} + \begin{pmatrix} -1 \\ 0 \end{pmatrix} = \begin{pmatrix} 4 \\ -6 \end{pmatrix}$$

$F = m\mathbf{a}$
$$8\mathbf{a} = \begin{pmatrix} 4 \\ -6 \end{pmatrix}$$
$$\mathbf{a} = \begin{pmatrix} 0.5 \\ -0.75 \end{pmatrix}$$



5 a $|\mathbf{a}| = \sqrt{0.5^2 + 0.75^2} = \sqrt{0.8125}$

$\tan\theta = \dfrac{0.75}{0.5}$

$\theta = 56°$

But bearing = 90° + θ (see diagram)

The acceleration has a magnitude of 0.901 m s^{-2} and acts at a bearing of 146°.

b $s = 20$, $u = 0$, $a = 0.901$

$s = ut + \frac{1}{2}at^2$

$20 = (0 \times t) + \left(\frac{1}{2} \times 0.901 \times t^2\right)$

$t^2 = \dfrac{20 \times 2}{0.901} = 44.39$

The particle takes 6.66 s to travel 20 m.

6 $\mathbf{R} = (2\mathbf{i} + 3\mathbf{j}) + (p\mathbf{i} + q\mathbf{j})$

Since $\mathbf{R}$ is parallel to $(-\mathbf{i} + 4\mathbf{j})$,

$\mathbf{R} = (-k\mathbf{i} + 4k\mathbf{j})$ where k is a constant

$(2\mathbf{i} + 3\mathbf{j}) + (p\mathbf{i} + q\mathbf{j}) = (-k\mathbf{i} + 4k\mathbf{j})$

Collecting **i** terms: $2 + p = -k$

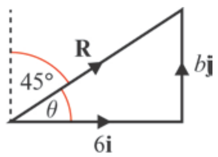
so $k = -2 - p$

Collecting **j** terms: $3 + q = 4k$

Substituting for k: $3 + q = 4(-2 - p)$

so $3 + q = -8 - 4p$

$4p + q + 11 = 0$



7 a $\theta = 90° - 45°$ (see diagram)

$\tan 45° = \dfrac{b}{6}$

$b = 6 \times \tan 45° = 6 \times 1$

The value of b is 6.

b $|\mathbf{R}| = \sqrt{6^2 + 6^2} = \sqrt{72}$

The magnitude of **R** is $6\sqrt{2}$ N (8.49 N to 3.s.f)

c $F = 6\sqrt{2}$, $m = 4$, $a = ?$

$F = ma$

$6\sqrt{2} = 4a$

The magnitude of the acceleration of the particle is $\dfrac{3\sqrt{2}}{2}$ m s^{-2} (2.12 m s^{-2} to 3 s.f.)

7 d $t = 5$, $u = 0$, $a = \frac{3\sqrt{2}}{2}$, $s = ?$

$$s = ut + \frac{1}{2}at^2$$

$$s = (0 \times 5) + \left(\frac{1}{2} \times \frac{3\sqrt{2}}{2} \times 5^2\right)$$

$$s = \frac{75\sqrt{2}}{4}$$

In the first 5 s the particle travels $\frac{75\sqrt{2}}{4}$ m (26.5 m to 3 s.f.).

8 a Since particle is in equilibrium, $\mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3 = 0$

$(-3\mathbf{i} + 7\mathbf{j}) + (\mathbf{i} - \mathbf{j}) + (p\mathbf{i} + q\mathbf{j}) = 0$
Collecting **i** terms: $-3 + 1 + p = 0$
Collecting **j** terms: $7 - 1 + q = 0$

The value of p is 2, and the value of q is −6.

b When $\mathbf{F}_2$ is removed, resultant force, $F = \mathbf{F}_1 + \mathbf{F}_3$
$F = (-3\mathbf{i} + 7\mathbf{j}) + (2\mathbf{i} - 6\mathbf{j}) = (-\mathbf{i} + \mathbf{j})$
The magnitude of this force is $\sqrt{1^2 + 1^2} = \sqrt{2}$

$s = 12$, $t = 10$, $u = 0$, $a = ?$

$$s = ut + \frac{1}{2}at^2$$

$$12 = (0 \times 20) + \left(\frac{1}{2} \times a \times 10^2\right)$$

$$12 = 50a$$

$$a = \frac{12}{50} = \frac{6}{25}$$

$$F = \sqrt{2},\ a = \frac{6}{25}$$

$$F = ma$$

$$\sqrt{2} = m \times \frac{6}{25}$$

The mass of the particle is $\frac{25\sqrt{2}}{6}$ kg.

9 Resultant force, $F = \mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3$

$F = (5\mathbf{i} + 6\mathbf{j}) + (2\mathbf{i} - 2\mathbf{j}) + (-\mathbf{i} - 4\mathbf{j}) = 6\mathbf{i}$
Since this has only a single component, the magnitude of the force is 6 N.
$a = 7$
$F = ma$
$6 = m \times 7$
$m = 6 \div 7$
The mass of the particle is 0.86 kg.

10 a $\mathbf{R} = \begin{pmatrix} 2 \\ 5 \end{pmatrix} + \begin{pmatrix} p \\ q \end{pmatrix}$

Since **R** is parallel to $\begin{pmatrix} 1 \\ -2 \end{pmatrix}$

$\mathbf{R} = \begin{pmatrix} k \\ -2k \end{pmatrix}$ where k is a constant

$\begin{pmatrix} 2 \\ 5 \end{pmatrix} + \begin{pmatrix} p \\ q \end{pmatrix} = \begin{pmatrix} k \\ -2k \end{pmatrix}$

Collecting **i** terms: $2 + p = k$

Collecting **j** terms: $5 + q = -2k$

Substituting for k: $5 + q = -2(2 + p)$

so $5 + q = -4 - 2p$

$2p + q + 9 = 0$

b $p = 1$

From **a** above, $k = 2 + p$

so $k = 2 + 1 = 3$

so $\mathbf{R} = \begin{pmatrix} k \\ -2k \end{pmatrix} = \begin{pmatrix} 3 \\ -6 \end{pmatrix}$

$|\mathbf{R}| = \sqrt{3^2 + (-6)^2} = \sqrt{45}$

$a = 15\sqrt{5}$, $F = \sqrt{45}$

$F = ma$

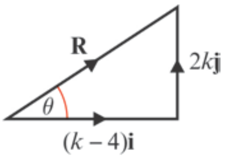
$\sqrt{45} = m \times 15\sqrt{5}$

$m = \frac{\sqrt{45}}{15\sqrt{5}} = \frac{\sqrt{9 \times 5}}{15\sqrt{5}} = \frac{3\sqrt{5}}{15\sqrt{5}} = \frac{1}{5} = 0.2$

The mass of the particle is 0.2 kg.

Challenge

Resultant force, $\mathbf{R} = \mathbf{F}_1 + \mathbf{F}_2$
$\mathbf{R} = -4\mathbf{i} + (k\mathbf{i} + 2k\mathbf{j})$



$F = ma$

$m = 0.5,\ a = 8\sqrt{17}$

So magnitude of the resultant force $= 0.5 \times 8\sqrt{17} = 4\sqrt{17}$

$|\mathbf{R}|^2 = (k-4)^2 + (2k)^2$

$\left(4\sqrt{17}\right)^2 = 16 \times 17 = k^2 - 8k + 16 + 4k^2$

$272 = 5k^2 - 8k + 16$

$5k^2 - 8k - 256 = 0$

$$k = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

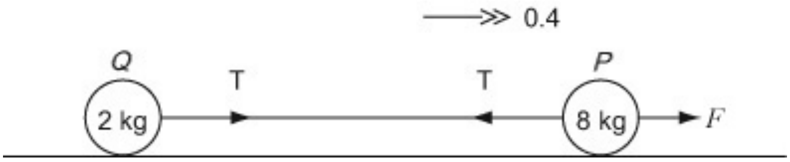
$$k = \frac{8 \pm \sqrt{8^2 - 4 \times 5 \times (-256)}}{2 \times 5} = \frac{8 \pm \sqrt{5184}}{10} = \frac{8 \pm 72}{10}$$

$k = -6.4$ or 8

Since k is given as a positive constant, the value of k is 8.

Forces and motion 10E

1



a $R(\rightarrow)$, $F = (2+8)\times 0.4$
$= 4$

Hence F is 4 N.

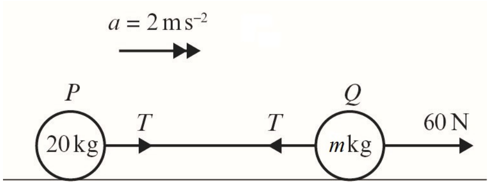
b For Q:

$R(\rightarrow)$, $T = 2\times 0.4$
$= 0.8$

The tension in the string is 0.8 N.

c Treating the string as inextensible (i.e. it does not stretch) allows us to assume that the acceleration of both masses is the same. Treating the string as light (i.e. having no/negligible mass) allows us to assume that the tension is the same throughout the length of the string and that its mass does not need to be considered when treating the system as a whole.

2

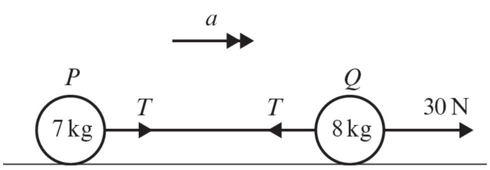


$F = ma$

a For the whole system: $F = 60$, $m = 20 + m = 10$, $a = 2$
$60 = (20 + m) \times 2$
$20 + m = 60 \div 2$
$m = 30 - 20$
The mass of Q is 10 kg.

b For P: $F = T$, $m = 20$, $a = 2$
$T = 20 \times 2$
The tension in the string is 40 N.

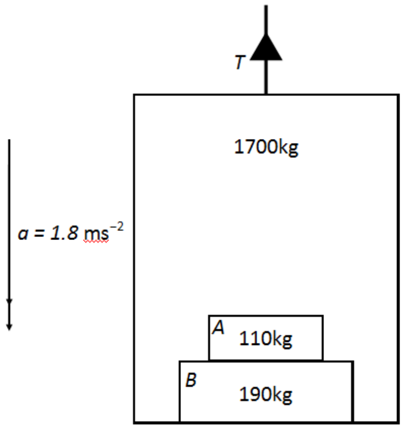
3 $F = ma$



3 a For the whole system: $F = 30$, $m = 8 + 7 = 15$, $a = ?$
$30 = 15a$
The acceleration of the system is 2 m s^{-2}.

b For P: $F = T$, $m = 7$, $a = 2$
$T = 7 \times 2$
The tension in the string is 14 N.

4



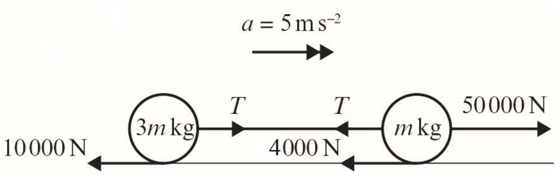
a Considering the system as a whole: total mass, m = 1700 + 110 + 190 = 2000 kg
Taking down as positive:
$F = ma = mg - T$
$2000 \times 1.8 = (2000 \times 9.8) - T$
$T = 19600 - 3600$
The tension in the cable is 16 000 N.

b i Force exerted on box A by box B is a normal reaction force, R_1 which acts upwards.
For box A, taking down as positive:
$110 \times 1.8 = 110g - R_1$
$R_1 = 110(g - 1.8)$
$R_1 = 110 \times 8$
Box B exerts an upwards force of 880 N on box A.

ii Let downward force exerted on lift by box B be S.
For lift alone, taking down as positive:
$1700 \times 1.8 = 1700g + S - T$
$S = T + 1700(1.8 - g)$
$S = 16\,000 - 13600 = 2400$

Alternatively (or as check), use Newton's third law of motion:
|Force exerted box B by box A| = |Force exerted on box A by box B| = 880 N
|Force exerted on lift by box B| = |Force exerted on box B by lift| = $|R_2|$
For box B, taking down as positive:
$190 \times 1.8 = 880 + 190g - R_2$
$R_2 = 880 + 190(g - 1.8)$
$R_2 = 880 + 1520 = 2400$

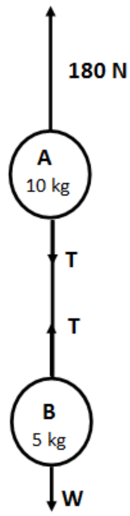
5 $F = ma$



a For the whole system:
$F = 50\,000 - 10\,000 - 4000 = 36\,000$
$a = 5$
total mass $= 3m + m = 4m$
$36\,000 = a \times$ total mass $= 4m \times 5 = 20m$
$m = 1800$
so $3m = 5400$
The mass of the lorry is 1800 kg, and that of the trailer is 5400 kg.

b For the trailer:
$F = T - 10\,000$, $m = 5400$, $a = 5$
$T - 10\,000 = 5400 \times 5 = 27\,000$
$T = 37\,000$
The tension in the tow-bar is 37 000 N.

c Treating the tow-bar as inextensible (i.e. it does not stretch) allows us to assume that the acceleration of the truck and the trailer are the same. Treating the tow-bar as light (i.e. having no/negligible mass) allows us to assume that the tension is the same throughout its length and that its mass does not need to be considered when treating the system as a whole.

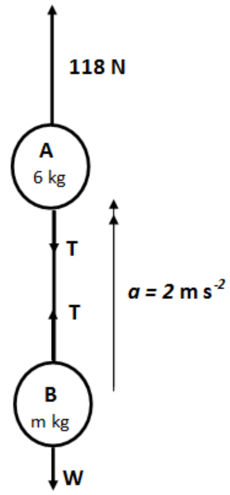


6 $F = ma$, $W = mg$
Taking upwards as positive

a For the whole system:
$180 - 15g = 15a$
$15a = 180 - (15 \times 9.8)$
$$a = \frac{180 - 147}{15} = 2.2$$
The acceleration is 2.2 m s^{-2}.

b For B:
$ma = T - W$
$5 \times 2.2 = T - (5 \times 9.8)$
$11 = T - 49$
The tension in the string is 60 N.

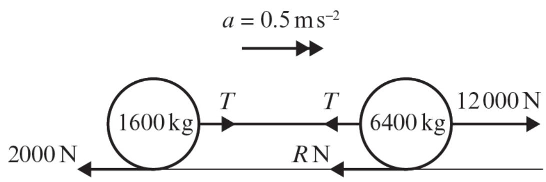
7 $F = ma,\ W = mg$
Taking up as positive



a For the whole system:
$118 - (6 + m)g = (6 + m) \times 2$
$118 = (6 + m)(2 + g) = (6 + m)(2 + 9.8)$
$\frac{118}{11.8} = 6 + m$
$10 = 6 + m$
The mass of B is 4 kg.

b For B:
$ma = T - W$
$4 \times 2 = T - (4 \times 9.8)$
$8 = T - 39.2$
The tension in the string is 47.2 N.

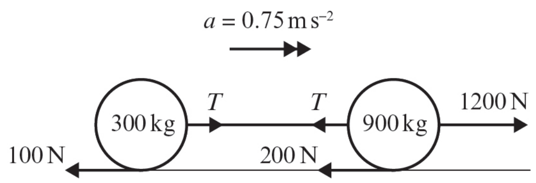
8 $F = ma$



a For the whole system:
$F = 12\,000 - 2000 - R$
$m = 1600 + 6400 = 8000$
$a = 0.5$
$10\,000 - R = 8000 \times 0.5 = 4000$
The resistance to the motion of the engine is 6000 N.

b For the carriage:
$F = T - 2000,\ m = 1600,\ a = 0.5$
$T - 2000 = 1600 \times 0.5 = 800$
The tension in the coupling is 2800 N.

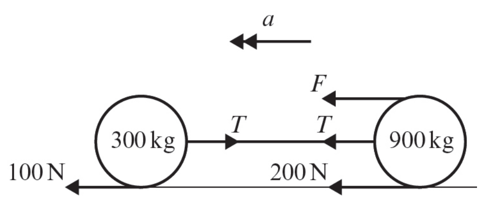
9 $F = ma$



a For the whole system:
$F = 1200 - 1000 - 200 = 900$
$m = 900 + 300 = 1200$
$900 = 1200a$
$a = 900 \div 1200 = 0.75$
The acceleration is $0.75\ \text{m s}^{-2}$, as required.

9 b For the trailer:
$F = T - 100$, $m = 300$, $a = 0.75$
$T - 100 = 300 \times 0.75 = 225$
The tension in the towbar is 325 N.

c



Taking ← as positive
Deceleration = α
Force on trailer = resistance to motion + thrust from tow-bar
Using $F = ma$
$100 + 100 = 300\alpha$

$$\alpha = \frac{200}{300} = \frac{2}{3}$$

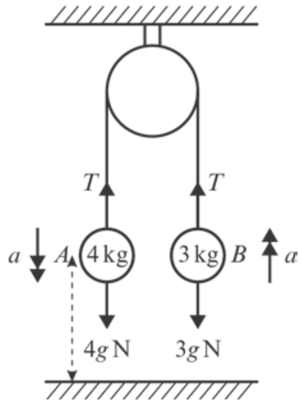
For car:
$F + 200 - 100 = 900\alpha$

$$F = \left(900 \times \frac{2}{3}\right) - 100 = 500$$

The force the brakes produce on the car is 500 N.

Forces and motion 10F

1 a



For A: $R(\downarrow)$, $4g - T = 4a$ (**1**)
For B: $R(\uparrow)$, $T - 3g = 3a$ (**2**)
$(\mathbf{1}) + (\mathbf{2}): 4g - 3g = 7a$

$\Rightarrow a = \frac{g}{7}$

Substituting into equation (**2**):

$T = 3a + 3g = \frac{3g}{7} + 3g = \frac{24g}{7}$
$= 33.6$ N (3 s.f.)

b $u = 0, a = \frac{g}{7}, s = 2, m, v = ?$
$v^2 = u^2 + 2as$
$v^2 = 0^2 + 2 \times \frac{g}{7} \times 2 = \frac{4g}{7} = 5.6$
$v = \sqrt{5.6} = 2.366...$

When A hits the ground it is travelling at 2.37 m s^{-1} (3 s.f.).

c For A: $(\downarrow)$

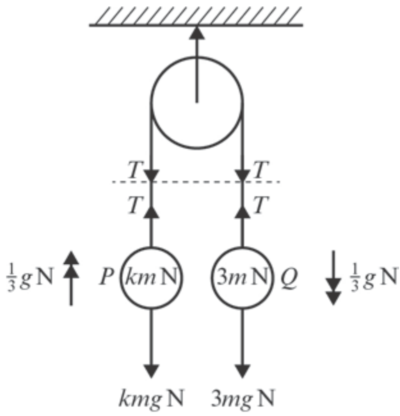
From part **b**, $v^2 = \frac{4g}{7}$

This represents the initial velocity of B when A hits the ground.

For B: $(\uparrow)$
$u^2 = \frac{4g}{7}, \ v = 0, \ a = -g, \ s = ?$
$v^2 = u^2 + 2as$
$0 = \frac{4g}{7} - 2gs \Rightarrow s = \frac{2}{7}$

The height above the initial position is $2\frac{2}{7}$ m.

2



a For Q, $R(\downarrow)$: $3mg - T = 3m \times \frac{1}{3}g = mg$

$2mg = T$

The tension in the string is $2mg$ N.

b For P, $R(\uparrow)$: $T - kmg = km \times \frac{1}{3}g$

$3T - 3kmg = kmg$
$3T = 4kmg$
Substituting for T: $6mg = 4kmg$

$$k = \frac{6mg}{4mg}$$

The value of k is 1.5.

c Because the pulley is smooth, there is no friction, so the magnitude of acceleration of P = the magnitude of acceleration of Q.

d Up is positive.
While Q is descending, the distance travelled by $P = s_1$

$u = 0,\ a = \frac{1}{3}g,\ t = 1.8,\ s = s_1$

$$s = ut + \frac{1}{2}at^2$$

$$s_1 = (0 \times 1.8) + \left(\frac{1}{2} \times \frac{g}{3} \times 1.8^2\right) = \frac{3.24g}{6} = 0.54g \qquad \textbf{(1)}$$

Speed of P at this time = v_1
Using $v^2 = u^2 + 2as$

After Q hits the ground, P travels freely under gravity and rises by a further distance s_2
$v = 0,\ u = v_1,\ a = -g,\ s = s_2$
$v^2 = u^2 + 2as$
$0^2 = 0.36g^2 - 2gs_2$

$$s_2 = \frac{0.36g^2}{2g} = 0.18g \qquad \textbf{(2)}$$

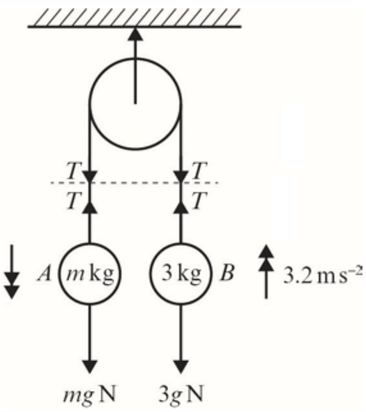
(**1**) + (**2**): Total distance travelled by P from its initial position = $s_1 + s_2$

2 d P and Q are at the same height initially, so P starts at height s_1 above the plane.
Its final position = initial position + total distance travelled

$$= s_1 + (s_1 + s_2) = 2s_1 + s_2 = 2 \times 0.54g + 0.18g = 1.26g$$

P reaches a maximum height of $1.26g$ m above the plane, as required.

3



a Since the pulley is smooth, |acceleration of A| = |acceleration of B|
For A: $s = 2.5$, $u = 0$, $t = 1.25$, $a = ?$ (down is positive)

$$s = ut + \tfrac{1}{2}at^2$$

$$2.5 = (0 \times 1.25) + \tfrac{1}{2}a \times 1.25^2$$

$$a = \frac{2.5 \times 2}{1.25^2} = 3.2$$

The initial acceleration of B is 3.2 m s^{-2} as required.

b For B, $R(\uparrow)$: $T - 3g = 3a$
$T = 3(a + g) = 3(3.2 + 9.8) = 39$
The tension in the string is 39 N.

c For A, $R(\downarrow)$: $mg - T = ma$
$T = m(g - a) = m(9.8 - 3.2) = 6.6m$
Substituting for T:
$39 = 6.6m$

$$m = \frac{39}{6.6} = \frac{390}{66} = \frac{65}{11} \text{ as required}$$

d Because the string is inextensible, the tension on both sides of the pulley is the same.

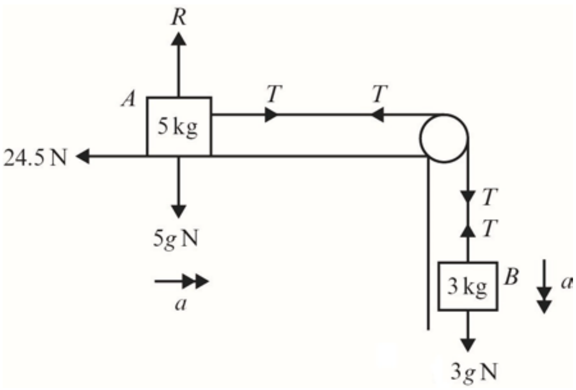
e The string will become taut again when B has risen to its maximum height and then descended to the point where A is just beginning to rise again.
If B reaches the maximum height t seconds after A hits the ground, it will also take t seconds to return to the same position as it is moving under gravity alone throughout this period. The total time of travel will be $2t$.
For B, taking up as positive, while the string is taut:
$u = 0$, $a = 1.4$, $s = 2.5$, m, $v = v_1$
$v^2 = u^2 + 2as$
$v_1^2 = 0^2 + 2 \times 3.2 \times 2.5 = 16$
Once the string is slack: $u = v_1 = 4$, $v = 0$, $a = -9.8$, $t = ?$
$v = u + at$
$0 = 4 - 9.8t$

3 e $t = \frac{4}{9.8} = \frac{40}{98} = \frac{20}{49}$

At this point B descends under gravity. After a further t seconds the string once again becomes taut.

The string becomes taut again $2t = \frac{40}{49}$ s after A hits the ground.

4



a For A: $R(\rightarrow)$, $T - 24.5 = 5a$ **(1)**

For B: $R(\downarrow)$, $3g - T = 3a$

$29.4 - T = 3a$ **(2)**

(1) + **(2)**: $29.4 - 24.5 = 8a$

$4.9 = 8a$

$0.6125 = a$

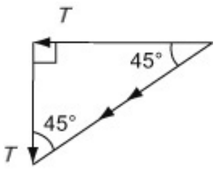
The acceleration of the system is 0.613 m s^{-2} (3 s.f.)

b $T - 24.5 = 5 \times 0.6125$

$T = 27.5625$

The tension in the string is 27.6 N (3 s.f.)

c



By Pythagoras,

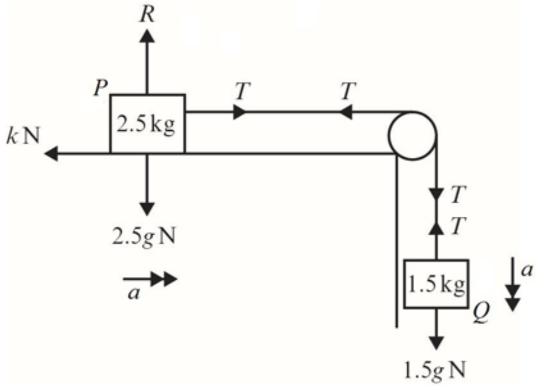
$F^2 = T^2 + T^2 = 2T^2$

$F = T\sqrt{2} = 27.5625 \times \sqrt{2}$

$= 38.979\ldots$

The magnitude of the force exerted on the pulley is 39 N (2 s.f.)

5



a **i** For Q: $s = 0.8$, $u = 0$, $t = 0.75$, $a = ?$ (down is positive)

$s = ut + \frac{1}{2}at^2$

$0.8 = (0 \times 0.75) + \frac{1}{2}a \times 0.75^2$

$a = \frac{0.8 \times 2}{0.75^2} = 2.844...$

The acceleration of Q is 2.84 m s^{-2} (3 s.f.)

ii For Q, $R(\downarrow)$: $1.5g - T = 1.5a$

$T = 1.5(g - a) = 1.5(9.8 - 2.84) = 10.44$

The tension in the string is 10.4 N (to 3 s.f.), as required.

iii For P, $R\ (\rightarrow)$: $T - k = 2.5a$

Substituting:

$10.4 - k = 2.5 \times 2.84$

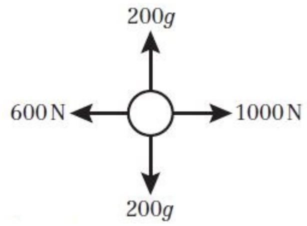
$k = 10.4 - 7.1$

The value of k is 3.3 N

b Because the string is inextensible, the tension in all parts of it is the same.

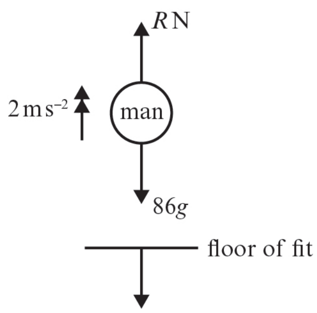
Forces and motion, Mixed exercise 10

1 a



b Vertical forces can be ignored as they are in equilibrium and at right angles to the direction of interest.
$F = ma$
$m = 200$, Resultant force, $F = 1000 - 200 - 400 = 400$
$400 = 200a$
The acceleration of the motorcycle is 2 m s^{-2}.

2



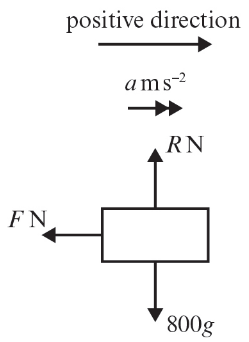
For the man

$$R(\uparrow), \quad R - 86g = 86 \times 2$$
$$R = 86 \times 9.8 + 86 \times 2$$
$$= 1014.8 \approx 1000$$

The reaction on the man on the floor is of equal magnitude to the action of the floor on the man and in the opposite direction.

The force that the man exerts on the floor of the lift is of magnitude 1000 N (2 s.f.) and acts vertically downwards.

3



3 **a** $u = 18,\ v = 12,\ t = 2.4,\ a = ?$

$$v = u + at$$
$$12 = 18 + 2.4a$$
$$a = \frac{12 - 18}{2.4} = -2.5$$

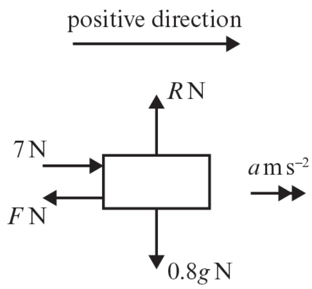
$$F = ma$$
$$-F = 800 \times -2.5 = -2000$$
$$F = 2000 \text{ N}$$

b $u = 18,\ v = 12,\ t = 2.4,\ s = ?$

$$s = \left(\frac{u + v}{2}\right)t$$
$$= \left(\frac{18 + 12}{2}\right) \times 2.4$$
$$= 15 \times 2.4 = 36$$

The distance moved by the car is 36 m

4



a $u = 2,\ v = 4,\ s = 4.8,\ a = ?$

$$v^2 = u^2 + 2as$$
$$4^2 = 2^2 + 9.6a$$
$$a = \frac{16 - 4}{9.6} = 1.25$$

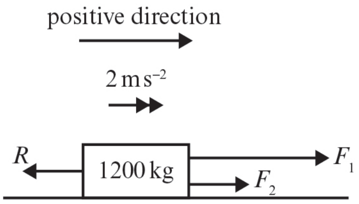
The magnitude of the acceleration of the block is 1.25 m s^{-2}

b $R(\uparrow),\quad F = ma = 0.8 \times 1.25 = 1$

$R(\rightarrow),\quad 7 - F = 6$

The magnitude of the frictional force between the block and the floor is 6 N.

5



Let R = the resistive force
Let F_1 = the driving force
Let F_2 = the resultant force

$F_2 = ma = 1200 \times 2 = 2400$

$F_1 = 3R \Rightarrow R = \frac{1}{3}F_1$

The driving force is the resultant force plus the resistive force:

$F_1 = R + F_2 = \frac{1}{3}F_1 + 2400$

$\frac{2}{3}F_1 = 2400$

$F_1 = 3600$

The magnitude of the driving force is 3600 N, as required.

6 $\mathbf{F}_1 = (3\mathbf{i} + 2\mathbf{j})$, $\mathbf{F}_2 = (4\mathbf{i} - \mathbf{j})$, $m = 0.25$

$F = \mathbf{F}_1 + \mathbf{F}_2 = ma$

$(3\mathbf{i} + 2\mathbf{j}) + (4\mathbf{i} - \mathbf{j}) = 0.25a$

$(7\mathbf{i} + \mathbf{j}) = 0.25a$

$a = \dfrac{(7\mathbf{i} + \mathbf{j})}{0.25}$

The acceleration is $(28\mathbf{i} + 4\mathbf{j})$ m s^{-2}.

7 $\mathbf{F}_1 = \begin{pmatrix} 2 \\ -1 \end{pmatrix}$ $\mathbf{F}_2 = \begin{pmatrix} 3 \\ -1 \end{pmatrix}$ $\mathbf{F}_3 = \begin{pmatrix} a \\ -2b \end{pmatrix}$ $m = 2,\ a = \begin{pmatrix} 3 \\ 2 \end{pmatrix}$

$F = \mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3 = ma$

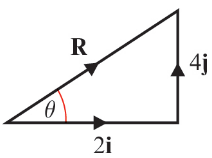
$\begin{pmatrix} 2 \\ -1 \end{pmatrix} + \begin{pmatrix} 3 \\ -1 \end{pmatrix} + \begin{pmatrix} a \\ -2b \end{pmatrix} = 2\begin{pmatrix} 3 \\ 2 \end{pmatrix} = \begin{pmatrix} 6 \\ 4 \end{pmatrix}$

Considering **i** components: $2 + 3 + a = 6$
$a = 6 - 5$

Considering **j** components: $-1 - 1 - 2b = 4$
$-2b = 4 + 2$

The values of a and b are 1 and −3, respectively.

8



a $|R| = \sqrt{2^2 + 4^2} = \sqrt{20} = 2\sqrt{5}$

Using $F = ma$

$2\sqrt{5} = 2a$

The acceleration of the sled is $\sqrt{5}$ m s^{-2}.

8 b $u = 0$, $t = 3$, $a = \sqrt{5}$, $s = ?$

$s = ut + \frac{1}{2}at^2$

$s = (0 \times 3) + \left(\frac{1}{2} \times \sqrt{5} \times 3^2\right) = \frac{9\sqrt{5}}{2}$

The sled travels a distance of $\frac{9\sqrt{5}}{2}$ m.

9 a Since object is in equilibrium, $\mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3 = 0$

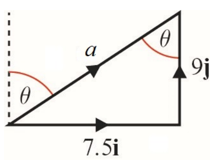
$(3a\mathbf{i} + 4b\mathbf{j}) + (5b\mathbf{i} + 2a\mathbf{j}) + (-15\mathbf{i} - 18\mathbf{j}) = 0$
Collecting **i** terms: $3a + 5b = 15$ **(1)**
Collecting **j** terms: $2a + 4b = 18$ **(2)**
Subtracting (**2**) from (**1**) gives $a + b = -3$
Therefore $b = -3 - a$

Substituting this into (**1**):
$3a + 5(-3 - a) = 15$
$3a - 15 - 5a = 15$
$-2a = 30$
$a = -15$
Substituting this into (**1**):
$3(-15) + 5b = 15$
$5b = 15 + 45 = 60$
$b = 12$

The values of a and b are -15 and 12, respectively.

b i $\mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3 = 0$, so when $\mathbf{F}_3$ is removed, the resultant force $F = -\mathbf{F}_3$
i.e. $F = (15\mathbf{i} + 18\mathbf{j})$

$m = 2$
$F = ma$
$(15\mathbf{i} + 18\mathbf{j}) = 2a$
$a = (7.5\mathbf{i} + 9\mathbf{j})$



$|a| = \sqrt{7.5^2 + 9^2} = \sqrt{137.25}$
Using Z angles (see diagram), bearing = θ
$\tan\theta = \frac{7.5}{9}$
The magnitude of the acceleration is 11.7 m s^{-2} and it has a bearing of 039.8° (both to 3 s.f.).

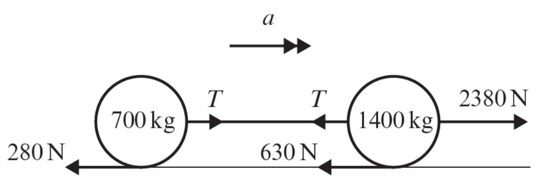
9 b ii $u = 0$, $t = 3$, $a = 11.7$, $s = ?$

$$s = ut + \frac{1}{2}at^2$$

$$s = (0 \times 3) + \left(\tfrac{1}{2} \times 11.7 \times 3^2\right) = \frac{105.3}{2}$$

The object travels a distance of 52.7 m (to 3 s.f.).

10



a $F = ma$

For the whole system:

$F = 2380 - 630 - 280 = 1470$

$m = 1400 + 700 = 2100$

$1470 = 2100a$

Since the tow-rope is inextensible, the acceleration of each part of the system is identical.

The acceleration of the car is 0.7 m s^{-2}.

b For the trailer:

$F = T - 280$, $m = 700$, $a = 0.7$

$T - 280 = 700 \times 0.7 = 490$

The tension in the tow-rope is 770 N.

c i For the car, after the rope breaks:

resultant force = 2380 – 630 = 1750

$m = 1400$

therefore $a = 1750 \div 1400 = 1.25$

$u = 12$

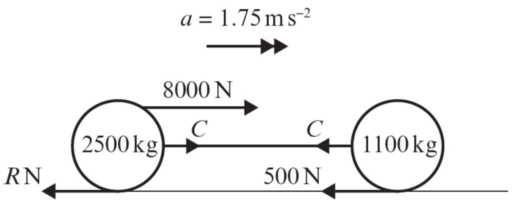
$$s = ut + \tfrac{1}{2}at^2$$

$$s = (12 \times 4) + \left(\tfrac{1}{2} \times 1.25 \times 4^2\right) = 48 + 10$$

In the first 4 s after the tow-rope breaks, the car travels 58 m.

ii Since the tow-rope is inextensible, the tension is constant throughout the length, and the acceleration of each part of the system is identical.

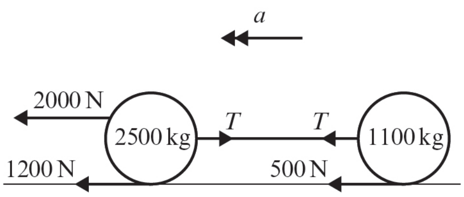
11



a $F = ma$
For the whole system:
$F = 8000 - 500 - R = 7500 - R$
$m = 2500 + 1100 = 3600$
$a = 1.75$
$7500 - R = 3600 \times 1.75 = 6300$
$R = 7500 - 6300$
The resistance to the motion of the train is 1200 N, as required.

b Considering the carriage only:
$C - 500 = 1100 \times 1.75 = 1925$
The compression force in the shunt is 2425 N.

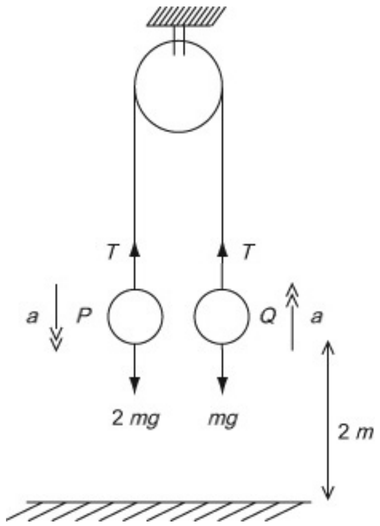
c



Taking ← as positive
Deceleration = α
Force on carriage = resistance to motion + thrust in shunt
Using $F = ma$
$500 + C = 1100\alpha$
$$\alpha = \frac{500 + C}{1100}$$
For engine:
$2000 + 1200 - C = 2500\alpha$
Substituting for α:
$$3200 - C = 2500 \times \left(\frac{500 + C}{1100}\right)$$
$$11\cancel{00}(3200 - C) = 25\cancel{00}(500 + C)$$
$$35200 - 11C = 12500 + 25C$$
$$35200 - 12500 = 11C + 25C$$
$$C = \frac{22700}{36}$$

The thrust in the shunt is 630 N (2 s.f.).

12 a



For P: $R(\downarrow)$, $2mg - T = 2ma$

For Q: $R(\uparrow)$, $T - mg = ma$

Add, $mg = 3ma$

$a = \frac{1}{3}g \text{ m s}^{-1}$

b For P:

$v^2 = u^2 + 2as$

$v^2 = 0 + 2 \times \frac{1}{3}g \times 2$

$v = \sqrt{\frac{4g}{3}}$

$= 3.6 \text{ m s}^{-1}$ (2 s.f.)

c For Q:

$R(\uparrow)$, $-mg = ma$

$a = -g$

$v^2 = u^2 + 2as$ $(\uparrow)$,

$0 = \frac{4g}{3} - 2gs$

$s = \frac{2}{3}\text{ m}$

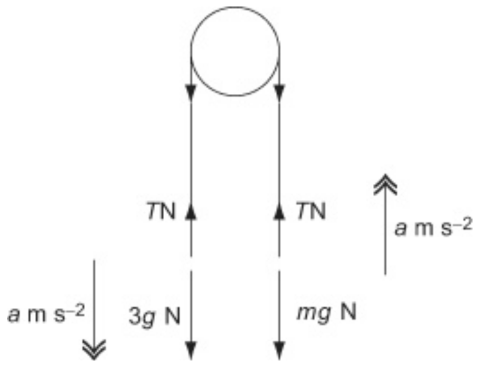
As Q is 4m above the ground when P stops moving $= 4 + s$

$\therefore$ Height above the ground $= 4\frac{2}{3}\text{ m}$

d i In an extensible string ⇒ acceleration of both masses is equal.

ii Smooth pulley ⇒ same tension in string either side of the pulley.

13 a



For the 3 kg mass

$R(\downarrow), \quad F = ma$

$3g - T = 3 \times \frac{3}{7} g$

$T = 3g - \frac{9}{7} g = \frac{12}{7} g$

The tension in the string is $\frac{12}{7} g$ N

b For the m kg mass

$R(\uparrow), \quad F = ma$

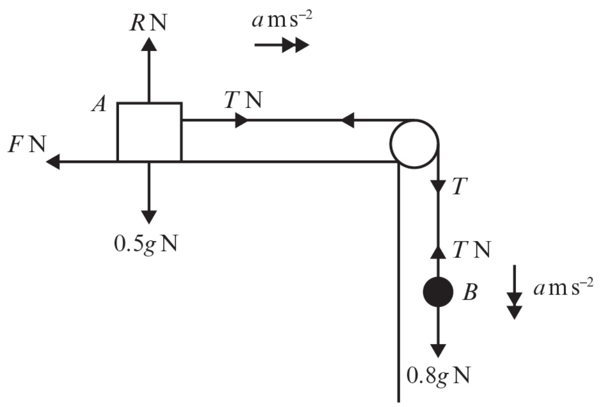
$T - mg = m \times \frac{3}{7} g$

Using the answer to **a**

$\frac{12}{7} g - mg = \frac{3}{7} mg$

$\frac{12}{7} = \frac{10}{7} m \Rightarrow m = 1.2$

14



a For B:

$u = 0, \; s = 0.4, \; t = 0.5, \; a = ?$

$s = ut + \frac{1}{2} at^2$

$0.4 = 0 + \frac{1}{2} a \times 0.5^2 = \frac{1}{8} a$

$a = 8 \times 0.4 = 3.2$

The acceleration of B is 3.2 m s^{-2}

14 b For B:

force = ma

$0.8g - T = 0.8 \times 3.2$

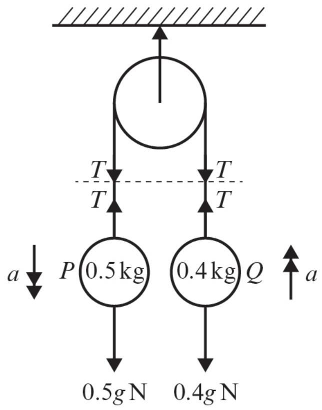
$T = 0.8 \times 9.8 - 0.8 \times 3.2$
$= 5.3$

The tension in the string is 5.3 N (2 s.f.). (As the numerical value $g = 9.8$ has been used, you should correct your answer to 2 significant figures.)

c $F = 3.7$ (2 s.f.)

d The information that the string is inextensible has been used in part **c** when the acceleration of A has been taken to be equal to the acceleration of B.

15



a i For P, $R(\downarrow)$: $0.5g - T = 0.5a$ **(1)**
ii For Q, $R(\uparrow)$: $T - 0.4g = 0.4a$ **(2)**

b (**1**) × 4: $2g - 4T = 2a$
(**2**) × 5: $5T - 2g = 2a$
Equating these:
$2g - 4T = 5T - 2g$
$9T = 4g$
The tension in the string is $\frac{4}{9}g$ N (4.35 N).

c Using equation (**1**):
$\frac{1}{2}g - \frac{4}{9}g = \frac{1}{2}a$
$g - \frac{8}{9}g = a$
The acceleration is $\frac{1}{9}g$ m s^{-2} (1.09 m s^{-2} (3 s.f.)).

15 d When the string breaks, Q has moved up a distance s_1 and reached a speed v_1
Now Q moves under gravity (after the string breaks) initially upwards.
To reach the floor it has to travel a distance $s = 2 + s_1$

While the string is intact, up positive:

$u = 0$, $t = 0.2$, $a = \frac{g}{9}$, $s_1 = ?$

$$s_1 = ut + \frac{1}{2}at^2$$
$$= (0\times 0.2) + \left(\frac{1}{2}\times\frac{g}{9}\times 0.2^2\right)$$
$$= \frac{g}{450}$$

$$v_1 = u + at$$
$$= 0 + \frac{g}{9}\times 0.2$$
$$= \frac{g}{45}$$

So, when the string breaks, Q is $2+\frac{g}{450}$ above the ground, a moving upwards with a speed of $\frac{g}{45}$.

After string breaks, Q moves under gravity. So taking down as positive, for the motion after the string breaks, we have

$u = v_1 = -\frac{g}{45}$, $a = g$, $s = 2+\frac{g}{450}$, $t = ?$

$$s = ut + \tfrac{1}{2}at^2$$
$$2+\frac{g}{450} = -\frac{g}{45}t + \frac{1}{2}gt^2$$
$$\frac{(900+g)}{450} = -\frac{g}{45}t + \frac{1}{2}gt^2$$
$$0 = \frac{1}{2}gt^2 - \frac{g}{45}t - \frac{(900+g)}{450}$$

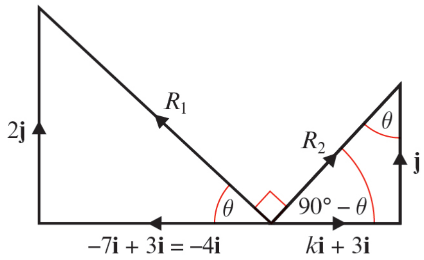
Let $g = 9.8 \Rightarrow 4.9t^2 - 0.21\dot{7}\dot{8}t - 2.02178 = 0$

$$t = \frac{-b\pm\sqrt{b^2-4ac}}{2a}$$
$$t = \frac{-0.2178\pm\sqrt{(-0.2178)^2-(4\times 4.9\times -2.02178)}}{2\times 4.9}$$
$$= \frac{-0.218\pm\sqrt{39.674}}{9.8}$$
$$= 0.66\text{ s or } -0.621\text{ s}$$

Only the positive root is relevant: $t = 0.66$ (2 s.f.)
Q hits the floor 0.66 s after the string breaks.

Challenge

Total force on first boat: $R_1 = (-7\mathbf{i} + 2\mathbf{j}) + 3\mathbf{i} = -4\mathbf{i} + 2\mathbf{j}$
Total force on second boat: $R_2 = (k\mathbf{i} + \mathbf{j}) + 3\mathbf{i} = (k + 3)\mathbf{i} + \mathbf{j}$
Since mass is a vector quantity, the acceleration of each boat will be parallel to the resultant force acting on it, so the relationship between the components of the accelerations is as shown in the diagram below.



From R_1: $\tan\theta = \frac{2}{4} = \frac{1}{2}$

From R_2: $\tan\theta = \frac{k+3}{1} = k+3$

Equating these: $\frac{1}{2} = k+3$

$2k + 6 = 1$

$2k = -5$

The value of k is -2.5.

Variable acceleration 11A

1 a $s = 9(1) - 1^3 = 8$ m

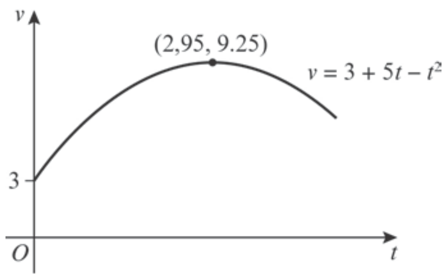
b $9t - t^3 = 0$
$t(9 - t^2) = 0$
Either $t = 0$ or $t^2 = 9$
$\Rightarrow t = 0$ or $t = \pm 3$

2 a At $t = 2$,
$s = 5(2)^2 - 2^3 = 12$
At $t = 4$,
$s = 5(4)^2 - 4^3 = 16$
Change in displacement = 16 − 12 = 4 m

b At $t = 3$,
$s = 5(3)^2 - 3^3 = 18$
Change in displacement in the third second = 18 − 12 = 6 m

3 a $v = 3 + 5(1) - 1^2 = 7$ m s^{-1}

b



At $t = 0$, $v = 3$
$v = 3$ again when $5t - t^2 = 0 \Rightarrow t = 5$

Using symmetry, turning point is when $t = 2.5$.
When $t = 2.5$,
$v = 3 + 5(2.5) - 2.5^2 = 9.25$
So in $0 \le t \le 4$, range of v is $3 \le v \le 9.25$
Greatest speed is 9.25 m s^{-1}.

c $v = 3 + 5(7) - 7^2$
$= 3 + 35 - 49$
$= -11$
When $t = 7$, the velocity of the particle is −11 m s^{-1}. This means it is moving in the opposite direction to that in which it was initially travelling.

4 a $s = 0$ when
$\frac{1}{5}(4t - t^2) = 0$
$\frac{1}{5}t(4 - t) = 0$
$\Rightarrow t = 0$ or $t = 4$
By symmetry, maximum distance is when $t = 2$.
When $t = 2$, $s = \frac{1}{5}(4(2) - 2^2)$
$= \frac{4}{5}$

4 a The maximum displacement is 0.8 m.

b When the toy car returns to P, $s = 0$

$\frac{1}{5}(4t - t^2) = 0$

$\frac{1}{5}t(4 - t) = 0$

$\Rightarrow t = 0$ or $t = 4$

The toy car returns to P after 4 s.

c The toy car travels to maximum distance and back again.

So total distance = 0.8 + 0.8 = 1.6 m

d The model is valid for $0 \leq t \leq 4$.

5 a When $t = 0$,

$v = 0 - 0 + 8 = 8$

The initial velocity is 8 m s^{-1}.

b $3t^2 - 10t + 8 = 0$

$(3t - 4)(t - 2) = 0$

The body is at rest when $t = \frac{4}{3}$ and $t = 2$.

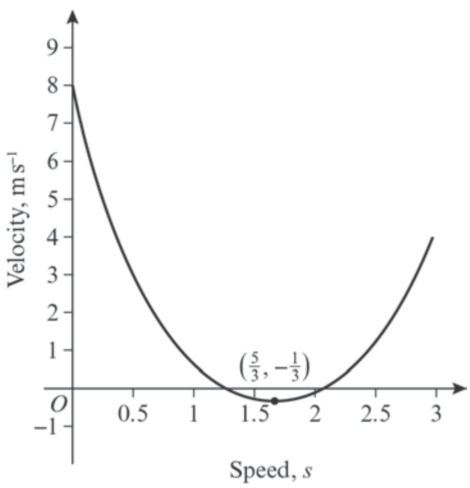
c $3t^2 - 10t + 8 = 5$

$3t^2 - 10t + 3 = 0$

$(3t - 1)(t - 3) = 0$

Velocity = 5 m s^{-1} when $t = \frac{1}{3}$ and $t = 3$.

d



Using the answer to part **b** and symmetry, the body has its maximum/minimum velocity when $t = \frac{5}{3}$ s.

When $t = \frac{5}{3}$,

$$v = 3\left(\frac{5}{3}\right)^2 - 10\left(\frac{5}{3}\right) + 8$$

$$= \frac{25}{3} - \frac{50}{3} + 8$$

$$= -\frac{25}{3} + 8$$

5 d $v = -\frac{1}{3}$

So in $0 \le t \le 2$, range of v is $-\frac{1}{3} \le v \le 8$.

Greatest speed is 8 m s^{-1}.

6 a $8t - 2t^2 = 0$

$2t(4 - t) = 0$

The particle is next at rest after 4 s.

b By symmetry, minimum/maximum velocity is when $t = 2$.

When $t = 2$,

$v = 8(2) - 2(2)^2$

$= 8$

So in $0 \le t \le 4$, greatest speed is 8 m s^{-1}.

7 $s = 3t^2 - t^3$

Model is valid until particle returns to starting point, i.e. until next point at which $s = 0$.

After this it would have a negative displacement, i.e. be beyond O.

$s = 0$ when

$3T^2 - T^3 = 0$

$T^2(3 - T) = 0$

$T \neq 0$ so $T = 3$

8 a $\frac{1}{5}\left(3t^2 - 10t + 3\right) = 0$

$3t^2 - 10t + 3 = 0$

$(3t - 1)(t - 3) = 0$

Particle is at rest when $t = \frac{1}{3}$ and $t = 3$.

b Using answer to part **a** and symmetry, the body has its maximum/minimum velocity when $t = \frac{5}{3}$.

When $t = \frac{5}{3}$,

$$v = \frac{1}{5}\left(3\left(\frac{5}{3}\right)^2 - 10\left(\frac{5}{3}\right) + 3\right)$$

$$= \frac{1}{5}\left(\frac{25}{3} - \frac{50}{3} + \frac{9}{3}\right)$$

$$= \frac{1}{5}\left(-\frac{16}{3}\right)$$

$$= -\frac{16}{15}$$

So in $0 \le t \le 3$, greatest speed is $\frac{16}{15} \text{ m s}^{-1}$.

Variable acceleration 11B

1 a $s = 4t^4 - \frac{1}{t}$

i $v = \frac{\mathrm{d}s}{\mathrm{d}t} = 16t^3 + \frac{1}{t^2}$

ii $a = \frac{\mathrm{d}v}{\mathrm{d}t} = 48t^2 - \frac{2}{t^3}$

b $x = \frac{2}{3}t^3 + \frac{1}{t^2}$

i $v = \frac{\mathrm{d}x}{\mathrm{d}t} = 2t^2 - \frac{2}{t^3}$

ii $a = \frac{\mathrm{d}v}{\mathrm{d}t} = 4t + \frac{6}{t^4}$

c $s = (3t^2 - 1)(2t + 5)$
$= 6t^3 + 15t^2 - 2t - 5$

i $v = \frac{\mathrm{d}s}{\mathrm{d}t} = 18t^2 + 30t - 2$

ii $a = \frac{\mathrm{d}v}{\mathrm{d}t} = 36t + 30$

d $x = \frac{3t^4 - 2t^3 + 5}{2t} = \frac{3t^3}{2} - t^2 + \frac{5}{2t}$

i $v = \frac{\mathrm{d}x}{\mathrm{d}t} = \frac{9t^2}{2} - 2t - \frac{5}{2t^2}$

ii $a = \frac{\mathrm{d}v}{\mathrm{d}t} = 9t - 2 + \frac{5}{t^3}$

2 a $x = 2t^3 - 8t$
$v = \frac{\mathrm{d}x}{\mathrm{d}t} = 6t^2 - 8$

When $t = 3$, $v = 6 \times 3^2 - 8 = 46$

The velocity of the particle when $t = 3$ is 46 m s^{-1}.

b $a = \frac{\mathrm{d}v}{\mathrm{d}t} = 12t$
When $t = 2$, $a = 12 \times 2 = 24$

The acceleration of the particle when $t = 2$ is 24 m s^{-2}.

3 P is at rest when $v = 0$.

$12 - t - t^2 = 0$
$(4 + t)(3 - t) = 0$
$t = -4$ or $t = 3$
$t \geq 0$, so $t = 3$

$a = \frac{dv}{dt} = -1 - 2t$

When $t = 3$, $a = -1 - 2 \times 3 = -7$

The acceleration of P when P is instantaneously at rest is -7 m s^{-2}, or 7 m s^{-2} in the direction of x decreasing.

4 $x = 4t^3 - 39t^2 + 120t$

$v = \frac{dx}{dt} = 12t^2 - 78t + 120$

P is at rest when $v = 0$.
$12t^2 - 78t + 120 = 0$
$2t^2 - 13t + 20 = 0$
$(2t - 5)(t - 4) = 0$

P is at rest when $t = 2.5$ and $t = 4$.

When $t = 2.5$, $x = 4(2.5)^3 - 39(2.5)^2 + 120(2.5) = 118.75$

When $t = 4$, $x = 4(4)^3 - 39(4)^2 + 120(4) = 112$

The distance between the two points where P is instantaneously at rest is $118.75 - 112 = 6.75$ m.

5 $v = kt - 3t^2$

a $a = \frac{dv}{dt} = k - 6t$

When $t = 0$, $a = 4$
$k - 6 \times 0 = 4$
$k = 4$

b P is at rest when $v = 0$.
$4t - 3t^2 = 0$
$t(4 - 3t) = 0$
P is at rest when $t = 0$ and $t = \frac{4}{3}$.

When $t = \frac{4}{3}$, $a = 4 - 6 \times \frac{4}{3} = 4 - 8 = -4$
When P is next at rest, the acceleration is -4 m s^{-2}.

6 $s = \frac{1}{4}(4t^3 - 15t^2 + 12t + 30)$

$v = \frac{ds}{dt} = \frac{1}{4}(12t^2 - 30t + 12)$

6 The print head is at rest when $v = 0$.

$\frac{1}{4}(12t^2 - 30t + 12) = 0$

$12t^2 - 30t + 12 = 0$

$2t^2 - 5t + 2 = 0$

$(2t - 1)(t - 2) = 0$

The print head is at rest when $t = 0.5$ and $t = 2$.

When $t = 0.5$,

$s = \frac{1}{4}(4(0.5)^3 - 15(0.5)^2 + 12(0.5) + 30)$

$= \frac{1}{4}(0.5 - 3.75 + 6 + 30)$

$= 8.1875$

When $t = 2$,

$s = \frac{1}{4}(4(2)^3 - 15(2)^2 + 12(2) + 30)$

$= \frac{1}{4}(32 - 60 + 24 + 30)$

$= 6.5$

Distance between these two points $= 8.1875 - 6.5$

$= 1.6875$ cm

$= 1.7$ cm (1 d.p.)

The distance between the points when the print head is instantaneously at rest is 1.7 cm.

Variable acceleration 11C

1 a $s = 0.4t^3 - 0.3t^2 - 1.8t + 5$

$v = \frac{ds}{dt} = 1.2t^2 - 0.6t - 1.8$

$\frac{dv}{dt} = 2.4t - 0.6$

$\frac{dv}{dt} = 0$ when $2.4t = 0.6$

$t = 0.25$

P is moving with minimum velocity at $t = 0.25$ s.

b When $t = 0.25$

$s = 0.4(0.25)^3 - 0.3(0.25)^2 - 1.8(0.25) + 5$

$= 4.54$ (3 s.f.)

When P is moving with minimum velocity, the displacement is 4.54 m.

c $v = \frac{ds}{dt} = 1.2t^2 - 0.6t - 1.8$

When $t = 0.25$, $v = 1.2 \times 0.25^2 - 0.6 \times 0.25 - 1.8$

$= -1.88$ (3 s.f.)

2 a $s = 4t^3 - t^4$

When $t = 4$,

$s = 4(4)^3 - 4^4 = 0$

The body returns to its starting position 4 s after leaving it.

b $s = 4t^3 - t^4 = s = t^3(4 - t)$

Since $t \geq 0$, t^3 is always positive.

Since $t \leq 4$, $(4 - t)$ is always positive.

So for $0 \leq t \leq 4$, s is always non-negative.

c $\frac{ds}{dt} = 12t^2 - 4t^3$

$\frac{ds}{dt} = 0$ when

$12t^2 - 4t^3 = 0$

$4t^2(3 - t) = 0$

$t = 0$ or 3

At $t = 0$, the body is at $s = 0$, so maximum displacement occurs when $t = 3$.

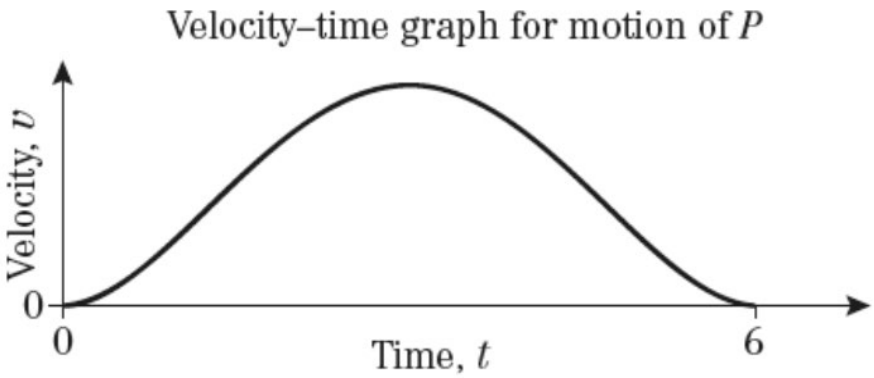
When $t = 3$, using factorised form of the equation of motion:

$s = 3^3(4 - 3) = 27$

The maximum displacement of the body from its starting point is 27 m.

3 a $v = t^2(6 - t)^2$

Velocity is zero when $t = 0$ and $t = 6$.
The graph touches the time axis at $t = 0$ and $t = 6$.
Graph only shown for $0 \le t \le 6$, as this is the range over which equation is valid.



b $v = t^2(6 - t)^2$
$= t^2(36 - 12t + t^2)$
$= 36t^2 - 12t^3 + t^4$

$$\frac{dv}{dt} = 72t - 36t^2 + 4t^3$$

$\frac{dv}{dt} = 0$ when
$72t - 36t^2 + 4t^3 = 0$
$4t(18 - 9t + t^2) = 0$
$4t(3 - t)(6 - t) = 0$

The turning points are at $t = 0$, $t = 3$ and $t = 6$.
$v = 0$ when $t = 0$ and $t = 6$, therefore the maximum velocity occurs when $t = 3$.

When $t = 3$,
$v = 3^2(6 - 3)^2 = 9 \times 9 = 81$

The maximum velocity is 81 m s^{-1} and the body reaches this 3 s after leaving O.

4 a $v = 2t^2 - 3t + 5$

For this particle to come to rest, v must be 0 for some positive value of t.

$2t^2 - 3t + 5 = 0$ must have real, positive roots.
$b^2 - 4ac = (-3)^2 - 4(2)(5)$
$= 9 - 40$
$= -31 < 0$

The equation therefore has no real roots, so v is never zero.

4 b $v = 2t^2 - 3t + 5$

$\frac{dv}{dt} = 4t - 3$

$\frac{dv}{dt} = 0$ when $4t = 3$

$t = 0.75$

Minimum velocity is when $t = 0.75$.

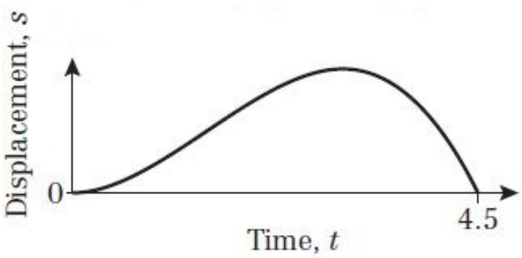
When $t = 0.75$, $v = 2(0.75)^2 - 3(0.75) + 5$
$= 1.125 - 2.25 + 5$
$= 3.875$
$= 3.88$ (3 s.f.)

The minimum velocity of the particle is 3.88 m s^{-1}.

5 a $s = \frac{9t^2}{2} - t^3$
$= t^2(4.5 - t)$

Displacement is zero when $t = 0$ and $t = 4.5$.
The graph touches the time axis at $t = 0$ and crosses it at $t = 4.5$.

Graph only shown for $0 \le t \le 4.5$, as this is range over which equation is valid.
The curve is cubic, so not symmetrical.



b For values of $t > 4.5$, s is negative. However s is a distance and can only be positive.

c $s = \frac{9t^2}{2} - t^3$

$\frac{ds}{dt} = 9t - 3t^2$

$\frac{ds}{dt} = 0$ when

$9t - 3t^2 = 0$
$3t(3 - t) = 0$

The turning points are at $t = 0$ and $t = 3$.
$s = 0$ when $t = 0$, so maximum distance occurs when $t = 3$.

When $t = 3$, using factorised form of the equation of motion:
$s = 3^2(4.5 - 3) = 9 \times 1.5 = 13.5$

The maximum distance of P from O is 13.5 m.

5 d $v = \frac{ds}{dt} = 9t - 3t^2$

$a = \frac{dv}{dt} = 9 - 6t$

When $t = 3$,

$a = 9 - 6 \times 3 = -9$

The magnitude of the acceleration of P at the maximum distance is 9 m s^{-2}.

6 $s = 3.6t + 1.76t^2 - 0.02t^3$

$\frac{ds}{dt} = 3.6 + 3.52t - 0.06t^2$

Maximum distance occurs when $\frac{ds}{dt} = 0$.

$\frac{ds}{dt} = 0$ when

$3.6 + 3.52t - 0.06t^2 = 0$

$3t^2 - 176t - 180 = 0$

$$t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{176 \pm \sqrt{(-176)^2 + (4)(3)(180)}}{2 \times 3}$$

$$= \frac{176 \pm \sqrt{33136}}{6}$$

$$= -1.005 \text{ or } 59.67$$

$t > 0$, so maximum distance occurs when $t = 59.67$.

When $t = 59.67$, $s = 3.6(59.67) + 1.76(59.67)^2 - 0.02(59.67)^3$

$= 2230$ (3 s.f.)

The maximum distance from the start of the track is 2230 m or 2.23 km.
Since this is less than 4 km, the train never reaches the end of the track.

Variable acceleration 11D

1 a $s = \int v\,dt$

$= \int (3t^2 - 1)\,dt$

$= t^3 - t + c$, where c is a constant of integration.

When $t = 0$, $x = 0$:

$0 = 0 - 0 + c \Rightarrow c = 0$

$s = t^3 - t$

b $s = \int v\,dt$

$= \int \left(2t^3 - \frac{3t^2}{2}\right)dt$

$= \frac{t^4}{2} - \frac{t^3}{2} + c$, where c is a constant of integration.

When $t = 0$, $x = 0$:

$0 = 0 - 0 + c \Rightarrow c = 0$

$s = \frac{t^4}{2} - \frac{t^3}{2}$

c $s = \int v\,dt$

$= \int (2\sqrt{t} + 4t^2)\,dt$

$= \frac{4}{3}t^{\frac{3}{2}} + \frac{4t^3}{3} + c$, where c is a constant of integration.

When $t = 0$, $x = 0$:

$0 = 0 + 0 + c \Rightarrow c = 0$

$s = \frac{4}{3}t^{\frac{3}{2}} + \frac{4t^3}{3}$

2 a $v = \int a\,dt$

$= \int (8t - 2t^2)\,dt$

$= 4t^2 - \frac{2t^3}{3} + c$, where c is a constant of integration.

When $t = 0$, $v = 0$:

$0 = 0 - 0 + c \Rightarrow c = 0$

$v = 4t^2 - \frac{2t^3}{3}$

b $v = \int a\,dt$

$= \int \left(6 + \frac{t^2}{3}\right)dt$

2 b $v = 6t + \frac{t^3}{9} + c$, where c is a constant of integration.

When $t = 0$, $v = 0$:
$0 = 0 + 0 + c \Rightarrow c = 0$
$v = 6t + \frac{t^3}{9}$

3 $x = \int v \, dt$
$= \int (8 + 2t - 3t^2) \, dt$
$= 8t + t^2 - t^3 + c$, where c is a constant of integration.

When $t = 0$, $x = 4$:
$4 = 0 + 0 - 0 + c \Rightarrow c = 4$
$x = 8t + t^2 - t^3 + 4$

When $t = 1$,
$x = 8 + 1 - 1 + 4 = 12$

The distance of P from O when $t = 1$ is 12 m.

4 a $v = \int a \, dt$
$= \int (16 - 2t) \, dt$
$= 16t - t^2 + c$, where c is a constant of integration.

When $t = 0$, $v = 6$:
$6 = 0 - 0 + c \Rightarrow c = 6$
$v = 16t - t^2 + 6$

b $x = \int v \, dt$
$= \int (16t - t^2 + 6) \, dt$
$= 8t^2 - \frac{t^3}{3} + 6t + k$, where k is a constant of integration.

When $t = 3$, $x = 75$:
$75 = 8 \times 3^2 - \frac{3^3}{3} + 6 \times 3 + k$
$\Rightarrow k = 75 - 72 + 9 - 18 = -6$
$x = 8t^2 - \frac{t^3}{3} + 6t - 6$

4 b When $t = 0$,
$x = 0 - 0 + 0 - 6 = -6$

5 $v = 6t^2 - 51t + 90$
P is at rest when $v = 0$.
$6t^2 - 51t + 90 = 0$

$2t^2 - 17t + 30 = 0$

$(2t - 5)(t - 6) = 0$

P is at rest when $t = 2.5$ and when $t = 6$.

$$s = \int_{2.5}^{6} (6t^2 - 51t + 90)\,dt$$
$$= \left[2t^3 - \frac{51t^2}{2} + 90t\right]_{2.5}^{6}$$
$$= \left(2\times6^3 - \frac{51\times6^2}{2} + 90\times6\right) - \left(2\times2.5^3 - \frac{51\times2.5^2}{2} + 90\times2.5\right)$$
$$= (432 - 918 + 540) - (31.25 - 159.375 + 225)$$
$$= -42.875\ldots$$
$$= -42.9 \text{ (3 s.f.)}$$

The negative sign indicates that the displacement is negative, but this can be ignored as distance is required.

The distance between the two points where P is at rest is 42.9 m (3 s.f.).

6 $s = \int v\,dt$

$$= \int (12 + t - 6t^2)\,dt$$
$$= 12t + \frac{t^2}{2} - 2t^3 + c$$, where c is a constant of integration.

When $t = 0$, $s = 0$:
$0 = 0 + 0 - 0 + c \Rightarrow c = 0$

$$s = 12t + \frac{t^2}{2} - 2t^3$$

$v = 0$ when
$12 + t - 6t^2 = 0$
$(3 - 2t)(4 + 3t) = 0$
$t > 0$, so $t = 1.5$

When $t = 1.5$, $s = 12\times1.5 + \frac{1.5^2}{2} - 2\times1.5^3$
$= 12.375\ldots$
$= 12.4$ (3 s.f.)

The distance of P from O when $v = 0$ is 12.4 m.

7 a $v = 4t - t^2$

P is at rest when $v = 0$.

$4t - t^2 = 0$

$t(4 - t) = 0$

$t > 0$, so $t = 4$

$$x = \int v\,dt$$
$$= \int (4t - t^2)\,dt$$
$$= 2t^2 - \frac{t^3}{3} + c$$, where c is a constant of integration.

When $t = 0$, $x = 0$

$0 = 0 + 0 + c \Rightarrow c = 0$

$$x = 2t^2 - \frac{t^3}{3}$$

When $t = 4$, $x = 2 \times 4^2 - \frac{4^3}{3}$

$= 10\frac{2}{3}$

b When $t = 5$, $x = 2 \times 5^2 - \frac{5^3}{3}$

$= 8\frac{1}{3}$

In the interval $0 \leq t \leq 5$, P moves to a point $10\frac{2}{3}$ m from O and then returns to a point $8\frac{1}{3}$ m from O.

The total distance moved is $10\frac{2}{3} + (10\frac{2}{3} - 8\frac{1}{3}) = 13$ m.

8 $$x = \int v\,dt$$
$$= \int (6t^2 - 26t + 15)\,dt$$
$= 2t^3 - 13t^2 + 15t + c$, where c is a constant of integration.

When $t = 0$, $x = 0$

$0 = 0 - 0 + 0 + c \Rightarrow c = 0$

$x = 2t^3 - 13t^2 + 15t$

$= t(2t^2 - 13t + 15)$

$= t(2t - 3)(t - 5)$

When $x = 0$ and t is non-zero, $t = 1.5$ or $t = 5$

P is again at O when $t = 1.5$ and $t = 5$.

9 a $$x = \int v\,dt$$
$$= \int (3t^2 - 12t + 5)\,dt$$
$= t^3 - 6t^2 + 5t + c$, where c is a constant of integration.

9 a When $t = 0$, $x = 0$

$0 = 0 - 0 + 0 + c \Rightarrow c = 0$

$x = t^3 - 6t^2 + 5t$

P returns to O when $x = 0$.

$t^3 - 6t^2 + 5t = 0$

$t(t^2 - 6t + 5) = 0$

$t(t - 1)(t - 5) = 0$

P returns to O when $t = 1$ and $t = 5$.

b $v = 0$ when

$3t^2 - 12t + 5 = 0$

$$t = \frac{12 \pm \sqrt{(-12)^2 - 4(3)(5)}}{6}$$

$= 0.473, 3.52$

So P does not turn round in the interval $2 \leq t \leq 3$.

When $t = 2$,

$x = 2^3 - 6 \times 2^2 + 5 \times 2$

$= 8 - 24 + 10$

$= -6$

When $t = 3$,

$x = 3^3 - 6 \times 3^2 + 5 \times 3$

$= 27 - 54 + 15$

$= -12$

The distance travelled by P in the interval $2 \leq t \leq 3$ is 6 m.

10 $v = \int a\,dt$

$= \int (4t - 3)\,dt$

$= 2t^2 - 3t + c$, where c is a constant of integration.

When $t = 0$, $v = 4$

$4 = 0 - 0 + c \Rightarrow c = 4$

$v = 2t^2 - 3t + 4$,

When $t = T$, $v = 4$ again

$4 = 2T^2 - 3T + 4$

$2T^2 - 3T = 0$

$T(2T - 3) = 0$

$T \neq 0$, so $T = 1.5$

11 a $v = \int a\,dt$

$= \int (t - 3)\,dt$

$= \frac{t^2}{2} - 3t + c$,

11 a When $t = 0$, $v = 4$

$4 = 0 - 0 + c \Rightarrow c = 4$

$$v = \frac{t^2}{2} - 3t + 4$$

b P is at rest when $v = 0$.

$$\frac{t^2}{2} - 3t + 4 = 0$$

$t^2 - 6t + 8 = 0$

$(t - 2)(t - 4) = 0$

$t = 2$ or $t = 4$

P is at rest when $t = 2$ and $t = 4$.

c
$$s = \int_2^4 \left(\frac{t^2}{2} - 3t + 4\right) dt$$
$$= \left[\frac{t^3}{6} - \frac{3t^2}{2} + 4t\right]_2^4$$
$$= \left(\frac{4^3}{6} - \frac{3 \times 4^2}{2} + 4 \times 4\right) - \left(\frac{2^3}{6} - \frac{3 \times 2^2}{2} + 4 \times 2\right)$$
$$= \left(\frac{32}{3} - 24 + 16\right) - \left(\frac{4}{3} - 6 + 8\right)$$
$$= \frac{8}{3} - \frac{10}{3}$$
$$= -\tfrac{2}{3}$$

The negative sign indicates that the displacement is negative, but this can be ignored as distance is required. The distance between the two points where P is at rest is $\tfrac{2}{3}$ m.

12
$$v = \int a\,dt$$
$$= \int (6t + 2)\,dt$$
$= 3t^2 + 2t + c$, where c is a constant of integration.

$$s = \int v\,dt$$
$$= \int (3t^2 + 2t + c)\,dt$$
$= t^3 + t^2 + ct + k$, where k is a constant of integration.

When $t = 2$, $s = 10$

$10 = 2^3 + 2^2 + 2c + k$

$2c + k = -2$ **(1)**

When $t = 3$, $s = 38$

$38 = 3^3 + 3^2 + 3c + k$

$3c + k = 2$ **(2)**

(2) − **(1)**:

$c = 4$

12 Substituting $c = 4$ into (**1**):
$2 \times 4 + k = -2$
$k = -10$

So the equations are:
$v = 3t^2 + 2t + 4$
$s = t^3 + t^2 + 4t - 10$

a When $t = 4$
$s = 4^3 + 4^2 + 4 \times 4 - 10$
$= 64 + 16 + 16 - 10$
$= 86$

When $t = 4$ s the displacement is 86 m.

b When $t = 4$
$v = 3 \times 4^2 + 2 \times 4 + 4$
$= 48 + 8 + 4$
$= 60$

When $t = 4$ s the velocity is 60 m s^{-1}.

Challenge

At $t = k$, the velocity given by both equations is identical, so:

$$\frac{k^2}{2}+2 = 10+\frac{k}{3}-\frac{k^2}{12}$$

$$6k^2 + 24 = 120 + 4k - k^2$$

$$7k^2 - 4k - 96 = 0$$

$$k = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

$$= \frac{4 \pm \sqrt{4^2+4\times7\times96}}{2\times7}$$

$$= \frac{4 \pm 52}{14}$$

$k > 0$, so $k = 4$

For first part of the motion, up to $t = 4$, $s = s_1$

$$s_1 = \int_0^4 \left(\frac{t^2}{2}+2\right)dt$$

$$= \left[\frac{t^3}{6}+2t\right]_0^4$$

$$= \left(\frac{4^3}{6}+2\times4\right)-\left(\frac{0^3}{6}+0\times4\right)$$

$$= \frac{56}{3}$$

For second part of the motion, from $t = 4$ to $t = 10$, $s = s_2$

$$s_2 = \int_4^{10} \left(10+\frac{t}{3}-\frac{t^2}{12}\right)dt$$

$$= \left[10t+\frac{t^2}{6}-\frac{t^3}{36}\right]_4^{10}$$

$$= \left(10\times10+\frac{10^2}{6}-\frac{10^3}{36}\right)-\left(10\times4+\frac{4^2}{6}-\frac{4^3}{36}\right)$$

$$= \frac{800}{9}-\frac{368}{9}$$

$$= 48$$

Total distance $= s_1 + s_2$

$$= \frac{56}{3} + 48$$

$$= \frac{200}{3}$$

The total distance travelled by the arm is $\frac{200}{3}$ m.

Variable acceleration 11E

1 $v = \int a\,\mathrm{d}t$

$= at + c$, where c is a constant of integration.

When $t = 0$, $v = 0$

$0 = a \times 0 + c \Rightarrow c = 0$

$v = at$

$s = \int v\,\mathrm{d}t$

$= \int at\,\mathrm{d}t$

$= \frac{1}{2}at^2 + k$, where k is a constant of integration.

When $t = 0$, $s = x$

$x = \frac{1}{2} \times a \times 0^2 + k \Rightarrow k = x$

$s = \frac{1}{2}at^2 + x$

2 **a** $v = \int a\,\mathrm{d}t$

$= \int 5\,\mathrm{d}t$

$= 5t + c$, where c is a constant of integration.

When $t = 0$, $v = 12$

$12 = 0 + c \Rightarrow c = 12$

$v = 12 + 5t$

b $s = \int v\,\mathrm{d}t$

$= \int (12 + 5t)\,\mathrm{d}t$

$= 12t + \frac{5}{2}t^2 + k$, where k is a constant of integration.

When $t = 0$, $s = 7$

$7 = 0 + 0 + k \Rightarrow k = 7$

$s = 12t + \frac{5}{2}at^2 + 7$

$= 12t + 2.5t^2 + 7$

3 $s = ut + \frac{1}{2}at^2$

$v = \frac{\mathrm{d}s}{\mathrm{d}t} = u + at$

$a = \frac{\mathrm{d}v}{\mathrm{d}t} = a$

So acceleration is constant.

4 A $s = 2t^2 - t^3$

$v = \frac{\mathrm{d}s}{\mathrm{d}t} = 4t - 3t^2$

4 A $a = \dfrac{dv}{dt} = 4 - 6t$

Not constant

B $s = 4t + 7$

$v = \dfrac{ds}{dt} = 4$

$a = \dfrac{dv}{dt} = 0$

No acceleration

C $s = \dfrac{t^2}{4}$

$v = \dfrac{ds}{dt} = \dfrac{t}{2}$

$a = \dfrac{dv}{dt} = \dfrac{1}{2}$

Constant acceleration

D $s = 3t - \dfrac{2}{t^2}$

$v = \dfrac{ds}{dt} = 3 + \dfrac{4}{t^3}$

$a = \dfrac{dv}{dt} = -\dfrac{12}{t^4}$

Not constant

E $s = 6$

$v = \dfrac{ds}{dt} = 0$

Particle stationary

5 **a** $v = u + at$

$u = 5,\ v = 13,\ t = 2$

$13 = 5 + 2a$

$a = \dfrac{13-5}{2} = 4$

The acceleration of the particle is 4 m s^{-2}.

b $v = \int a\,dt$

$= \int 4\,dt$

$= 4t + c$, where c is a constant of integration.

5 b When $t = 0$, $v = 5$
$5 = 0 + c \Rightarrow c = 5$
$v = 4t + 5$
$s = \int v\,dt$
$= \int (4t + 5)\,dt$
$= 2t^2 + 5t + k$, where k is a constant of integration.

When $t = 0$, $s = 0$
$0 = 0 + 0 + k \Rightarrow k = 0$
$s = 2t^2 + 5t$

This is an equation of the required form with $p = 2$, $q = 5$ and $r = 0$.

6 a $s = 25t - 0.2t^2$
When $t = 40$, $s = 25 \times 40 - 0.2 \times 40^2$
$= 680$

The distance AB is 680 m.

b $v = \frac{ds}{dt} = 25 - 0.4t$

$a = \frac{dv}{dt} = -0.4$

The train has a constant acceleration (of -0.4 m s^{-2}).

c Taking the direction in which the train travels to be positive:
For the bird: $a = -0.6$, $u = -7$, initial displacement = 680
$v_B = \int a\,dt$
$= \int -0.6\,dt$
$= -0.6t + c$, where c is a constant of integration.

When $t = 0$, $v_B = -7$
$-7 = 0 + c \Rightarrow c = -7$
$v = -0.6t - 7$
$s_B = \int v_B\,dt$
$= \int (-0.6t - 7)\,dt$
$= -0.3t^2 - 7t + k$, where k is a constant of integration.

When $t = 0$, $s_B = 680$
$680 = 0 - 0 + k \Rightarrow k = 680$
$s_B = -0.3t^2 - 7t + 680$

When the bird is directly above the train, the displacement of both train and bird are the same.

$25t - 0.2t^2 = -0.3t^2 - 7t + 680$
$0.1t^2 + 32t - 680 = 0$
$t^2 + 320t - 6800 = 0$
$(t - 20)(t + 340) = 0$

6 **c** $t > 0$, so $t = 20$

When $t = 20$,
$s = 25 \times 20 - 0.2 \times 20^2$
$= 420$

The bird is directly above the train 420 m from A.

Variable acceleration, Mixed Exercise 11

1 a $v = 15 - 3t$

P is at rest when $v = 0$.

$0 = 15 - 3t$

$t = 5$

b $s = \int_0^5 v\,\mathrm{d}t$

$= \int_0^5 (15 - 3t)\,\mathrm{d}t$

$= \left[15t - \frac{3t^2}{2}\right]_0^5$

$= \left(15 \times 5 - \frac{3 \times 5^2}{2}\right) - 0$

$= 37.5$

The distance travelled by P is 37.5 m.

2 a $v = 6t + \frac{1}{2}t^3$

$a = \frac{\mathrm{d}v}{\mathrm{d}t}$

$= 6 + \frac{3}{2}t^2$

When $t = 4$, $a = 6 + \frac{3}{2} \times 4^2$

$= 30$

The acceleration of P when $t = 4$ is 30 m s^{-2}.

b $x = \int v\,\mathrm{d}t$

$= \int (6t + \frac{1}{2}t^3)\,\mathrm{d}t$

$= 3t^2 + \frac{t^4}{8} + k$, where k is a constant of integration.

When $t = 0$, $x = -5$

$-5 = 0 + 0 + k \Rightarrow k = -5$

$x = 3t^2 + \frac{t^4}{8} - 5$

When $t = 4$, $x = 3 \times 4^2 + \frac{4^4}{8} - 5$

$= 75$

$OP = 75$ m

3 a $v = \int a\,\mathrm{d}t$
$= \int (2 - 2t)\,\mathrm{d}t$
$= 2t - t^2 + c$, where c is a constant of integration.

When $t = 0$, $v = 8$
$8 = 0 - 0 + c \Rightarrow c = 8$
$v = 2t - t^2 + 8$

Let s m be the displacement from A at time t seconds.
$s = \int v\,\mathrm{d}t$
$= \int (2t - t^2 + 8)\,\mathrm{d}t$
$= t^2 - \frac{t^3}{3} + 8t + k$, where k is a constant of integration.

When $t = 0$, $s = 0$
$0 = 0 - 0 + 0 + k \Rightarrow k = 0$

Displacement of P from A at time t seconds $= t^2 - \frac{t^3}{3} + 8t$

b The greatest positive displacement of P occurs when $\frac{\mathrm{d}s}{\mathrm{d}t} = v = 0$:

$2t - t^2 + 8 = 0$
$t^2 - 2t - 8 = 0$
$(t + 2)(t - 4) = 0$
$t > 0$, so $t = 4$

When $t = 4$, $s = 4^2 - \frac{4^3}{3} + 8 \times 4$
$= 26\frac{2}{3} < 30$

Hence, P does not reach B.

c P returns to A when $s = 0$.

$t^2 - \frac{t^3}{3} + 8t = 0$
$t^3 - 3t^2 - 24t = 0$
$t(t^2 - 3t - 24) = 0$

$$t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$= \frac{3 \pm \sqrt{(-3)^2 - 4(1)(-24)}}{2(1)}$$
$$= \frac{3 \pm \sqrt{105}}{2}$$

$t > 0$, so $t = 6.62$

P returns to A when $t = 6.62$.

3 d Distance between two instants when P passes through A = 2 × maximum distance found in part **b**

$= 2 \times \frac{80}{3}$

$= \frac{160}{3}$.

Total distance travelled by P between the two instants when it passes through A is $\frac{160}{3}$ m.

4 a $a = \frac{\mathrm{d}v}{\mathrm{d}t}$ so speed has maximum value when $a = 0$.

$0 = 8 - 2t^2$

$2t^2 = 8$

$t^2 = 4$

$t > 0$, so $t = 2$

$v = \int a\,\mathrm{d}t$

$= \int (8 - 2t^2)\,\mathrm{d}t$

$= 8t - \frac{2t^3}{3} + c$, where c is a constant of integration.

When $t = 0$, $v = 0$

$0 = 0 - 0 + c \Rightarrow c = 0$

$v = 8t - \frac{2t^3}{3}$

When $t = 2$, $v = (8 \times 2) - \frac{2 \times 2^3}{3}$

$= 16 - \frac{16}{3}$

$= \frac{32}{3}$

The greatest positive speed of the particle is $\frac{32}{3}$ m s^{-1}.

b $s = \int v\,\mathrm{d}t$

$= \int \left(8t - \frac{2t^3}{3}\right)\mathrm{d}t$

$= 4t^2 - \frac{t^4}{6} + k$, where k is a constant of integration.

When $t = 0$, $s = 0$

$0 = 0 - 0 + k \Rightarrow k = 0$

$s = 4t^2 - \frac{t^4}{6}$

When $t = 2$, $s = 4 \times 2^2 - \frac{2^4}{6}$

$= 16 - \frac{16}{6}$

$= \frac{40}{3}$

The distance covered by the particle during the first two seconds is $\frac{40}{3}$ m.

5 **a** $s = -t^3 + 11t^2 - 24t$

$v = \frac{ds}{dt}$

$= -3t^2 + 22t - 24 \text{ m s}^{-1}$

b P is at rest when $v = 0$.

$-3t^2 + 22t - 24 = 0$

$3t^2 - 22t + 24 = 0$

$(3t - 4)(t - 6) = 0$

$t = \frac{4}{3}$ or $t = 6$

P is at rest when $t = \frac{4}{3}$ and $t = 6$.

c $a = \frac{dv}{dt}$

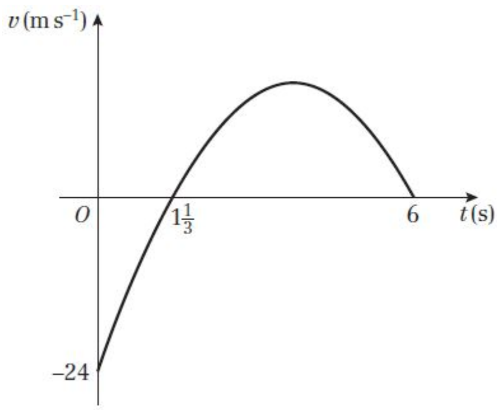
$= -6t + 22$

$a = 0$ when $0 = -6t + 22$

$t = \frac{11}{3}$

The acceleration is zero when $t = \frac{11}{3}$.

d



e The speed of P is 16 when $v = 16$ and $v = -16$.

When $v = 16$, $-3t^2 + 22t - 24 = 16$

$3t^2 - 22t + 40 = 0$

$(3t - 10)(t - 4) = 0$

$t = \frac{10}{3}$ or $t = 4$

When $v = -16$, $-3t^2 + 22t - 24 = -16$

$3t^2 - 22t + 8 = 0$

$$t = \frac{22 \pm \sqrt{22^2 - 4 \times 3 \times 8}}{2 \times 3}$$

$$= \frac{22 \pm \sqrt{388}}{6}$$

$= 0.384$ or 6.95

From the graph in part **d**, the required values are $0 \le t < 0.384$, $\frac{10}{3} < t < 4$.

6 a The body is at rest when $v = 0$.
$3t^2 - 11t + 10 = 0$
$(3t - 5)(t - 2) = 0$
$t = \frac{5}{3}$ or $t = 2$

The body is at rest when $t = \frac{5}{3}$ and $t = 2$.

b $a = \frac{dv}{dt}$
$= 6t - 11$

When $t = 4$, $a = (6 \times 4) - 11$
$= 24 - 11$
$= 13$

When $t = 4$, the acceleration is 13 m s^{-2}.

c From part **a**, the body changes direction when $t = \frac{5}{3}$ and $t = 2$.
s_1 = displacement for $0 \le t \le \frac{5}{3}$
s_2 = displacement for $\frac{5}{3} \le t \le 2$
s_3 = displacement for $2 \le t \le 4$

$$s_1 = \int_0^{\frac{5}{3}} (3t^2 - 11t + 10)\,dt$$
$$= \left[t^3 - \frac{11t^2}{2} + 10t\right]_0^{\frac{5}{3}}$$
$$= \left(\left(\frac{5}{3}\right)^3 - \frac{11 \times \left(\frac{5}{3}\right)^2}{2} + 10 \times \frac{5}{3}\right) - 0$$
$$= \frac{125}{27} - \frac{275}{18} + \frac{50}{3}$$
$$= \frac{325}{54}$$

$$s_2 = \int_{\frac{5}{3}}^{2} (3t^2 - 11t + 10)\,dt$$
$$= \left[t^3 - \frac{11t^2}{2} + 10t\right]_{\frac{5}{3}}^{2}$$
$$= \left(2^3 - \frac{11 \times 2^2}{2} + 10 \times 2\right) - \frac{325}{54}$$
$$= 6 - \frac{325}{54}$$
$$= -\frac{1}{54}$$

$$s_3 = \int_2^4 (3t^2 - 11t + 10)\,dt$$

6 c $s_3 = \left[t^3 - \frac{11t^2}{2} + 10t\right]_2^4$

$= 4^3 - \frac{11\times4^2}{2} + 10\times4 - 6$

$= 64 - 88 + 40 - 6$

$= 10$

Sign to be ignored when calculating distance:

Total distance $= s_1 + s_2 + s_3$

$= \frac{325}{54} + \frac{1}{54} + 10$

$= \frac{433}{27}$

The total distance travelled is 16.0 m (1 d.p.).

7 a $v = \int a\,\mathrm{d}t$

$= \int (2t^3 - 8t)\,\mathrm{d}t$

$= \frac{t^4}{2} - 4t^2 + c$, where c is a constant of integration.

When $t = 0$, $v = 6$

$6 = 0 - 0 + c \Rightarrow c = 6$

$6 = \frac{0^4}{2} - (4\times0^2) + c$

$v = \frac{t^4}{2} - 4t^2 + 6$

b $s = \int v\,\mathrm{d}t$

$= \int\left(\frac{t^4}{2} - 4t^2 + 6\right)\mathrm{d}t$

$= \frac{t^5}{10} - \frac{4t^3}{3} + 6t + k$, where k is a constant of integration.

When $t = 0$, $s = 0$

$0 = 0 - 0 + 0 + k \Rightarrow k = 0$

$s = \frac{t^5}{10} - \frac{4t^3}{3} + 6t$

c Particle is at rest when $v = 0$.

$\frac{t^4}{2} - 4t^2 + 6 = 0$

$t^4 - 8t^2 + 12 = 0$

$(t^2 - 2)(t^2 - 6) = 0$

$t \geq 0$, so $t = \sqrt{2}$ or $t = \sqrt{6}$

The particle is at rest when $t = \sqrt{2}$ and $t = \sqrt{6}$.

8 $x = \dfrac{t^4 - 12t^3 + 28t^2 + 400}{50}$

$\dfrac{dx}{dt} = \dfrac{4t^3 - 36t^2 + 56t}{50}$

Maxima and minima occur when $\dfrac{dx}{dt} = 0$.

$\dfrac{4t^3 - 36t^2 + 56t}{50} = 0$

$t^3 - 9t^2 + 14t = 0$

$t(t^2 - 9t + 14) = 0$

$t(t - 2)(t - 7) = 0$

So turning points are at $t = 0$, $t = 2$ and $t = 7$.

From the sketch graph, the drone is at a greater height when $t = 2$ than when $t = 0$, and $t = 7$ corresponds to the minimum height over the given interval.

When $t = 2$, $x = \dfrac{2^4 - (12 \times 2^3) + (28 \times 2^2) + 400}{50}$

$= \dfrac{16 - 96 + 112 + 400}{50}$

$= 8.64$

When $t = 7$, $x = \dfrac{7^4 - (12 \times 7^3) + (28 \times 7^2) + 400}{50}$

$= \dfrac{2401 - 4116 + 1372 + 400}{50}$

$= 1.14$

The maximum height reached by the drone is 8.64 m, and the minimum height is 1.14 m.

9 When $t = 0$, $v = u = 800$, $s = 1500$

When $t = 25$, $v = 0$

Using $v = u + at$:

$0 = 800 + 25a$

$a = \dfrac{-800}{25} = -32$

$v = \int a\,dt$

$= \int -32\,dt$

$= -32t + c$, where c is a constant of integration.

When $t = 0$, $v = 800$

$800 = 0 + c \Rightarrow c = 800$

$v = 800 - 32t$

$s = \int v\,dt$

$= \int (800 - 32t)\,dt$

9 $s = 800t - 16t^2 + k$, where k is a constant of integration.

When $t = 0$, $s = 1500$
$1500 = 0 - 0 + k \Rightarrow k = 1500$
$s = 800t - 16t^2 + 1500$
So $a = 1500$, $b = 800$, $c = -16$

10 a $v = \int a\,dt$
$= \int (20 - 6t)\,dt$
$= 20t - 3t^2 + c$, where c is a constant of integration.

When $t = 0$, $v = 7$
$7 = 0 - 0 + c \Rightarrow c = 7$
$v = 20t - 3t^2 + 7$
$= 7 + 20t - 3t^2$

b When $t = 7$, $v = 7 + 20 \times 7 - 3 \times 7^2$
$= 7 + 140 - 147$
$= 0$

P's maximum speed in the interval $0 \leq t \leq 7$ is when $\frac{dv}{dt} = 0$

$\frac{dv}{dt} = 20 - 6t$
$0 = 20 - 6t$
$t = \frac{10}{3}$

When $t = \frac{10}{3}$, $v = 7 + 20 \times \frac{10}{3} - 3 \times \left(\frac{10}{3}\right)^2$
$= \frac{121}{3}$

The greatest speed of P in the interval $0 \leq t \leq 7$ is $40\frac{1}{3}$ m s^{-1}.

c $s = \int_0^7 (7 + 20t - 3t^2)\,dt$
$= \left[7t + 10t^2 - t^3\right]_0^7$
$= 7 \times 7 + 10 \times 7^2 - 7^3 - 0$
$= 196$

The distance travelled by P in the interval $0 \leq t \leq 7$ is 196 m.

11 $a \propto (7 - t^2)$

So $a = k(7 - t^2)$

$= 7k - kt^2$

$$v = \int a\,dt$$
$$= \int (7k - kt^2)\,dt$$
$$= 7kt - \frac{kt^3}{3} + c$$, where c is a constant of integration.

When $t = 0$, $v = 0$

$0 = 0 - 0 + c \Rightarrow c = 0$

$$v = 7kt - \frac{kt^3}{3}$$

When $t = 3$, $v = 6$

$6 = 21k - 9k$

$12k = 6$

$k = \frac{1}{2}$

$$v = \frac{7t}{2} - \frac{t^3}{6}$$

$$s = \int v\,dt$$
$$= \int \left(\frac{7t}{2} - \frac{t^3}{6}\right) dt$$
$$= \frac{7t^2}{4} - \frac{t^4}{24} + d$$, where d is a constant of integration.

When $t = 0$, $v = 0$

$0 = 0 - 0 + d \Rightarrow d = 0$

$$s = \frac{7t^2}{4} - \frac{t^4}{24}$$
$$= \tfrac{1}{24}t^2(42 - t^2)$$

12 a Time cannot be negative so $t \geq 0$.

When $t = 0$, $s = 0$

When $t = 5$, $s = 5^4 - 10 \times 5^3 + 25 \times 5^2$

$= 625 - 1250 + 625$

$= 0$

So when $t = 5$, the mouse is again at a distance of zero from the hole: it has returned.

12 b $s = t^4 - 10t^3 + 25t^2$

When mouse is at the greatest distance, $\frac{ds}{dt} = 0$

$\frac{ds}{dt} = 4t^3 - 30t^2 + 50t$

When $\frac{ds}{dt} = 0$, $4t^3 - 30t^2 + 50t = 0$

$2t(2t^2 - 15t + 25) = 0$

$2t(2t - 5)(t - 5) = 0$

$s = 0$ when $t = 0$ and $t = 5$, so maximum is when $t = 2.5$.

When $t = 2.5$, $s = 2.5^4 - 10 \times 2.5^3 + 25 \times 2.5^2$
$= 39.1$

The greatest distance of the mouse from the hole is 39.1 m.

13 a Any two from:

As the shuttle rises, it burns large amounts of fuel, reducing mass and therefore allowing the same force to produce greater acceleration. (Hence positive terms in the equation.)

While the shuttle remains in the atmosphere, the air resistance forces on it will be changing in a complex way: the increasing speed will cause them to increase, but reduced density of the atmosphere at greater heights will reduce their effect.

At greater heights, the gravitational pull of the Earth is less, which increases the resultant force on the shuttle and increases the acceleration. (In practice, this effect is small compared to that of the mass reduction.)

As the fuel from each tank in the booster rockets is used up, they may become less efficient, reducing the thrust they produce. (The fuel feed mechanisms are designed to prevent this and ensure smooth transitions between each stage, but any astronaut can tell you that there is no such thing as a smooth journey into space!)

b $v = \int a\,dt$

$= \int ((6.7\times10^{-7})t^3 - (3.98\times10^{-4})t^2 + 0.105t + 0.859)\,dt$

$= (1.68 \times 10^{-7})t^4 - (1.33 \times 10^{-4})t^3 + 0.0525t^2 + 0.859t + c$, where c is a constant of integration.

When $t = 124$, $v = 974$
$974 = (1.68 \times 10^{-7})(124)^4 - (1.33 \times 10^{-4})(124)^3 + 0.0525(124)^2 + 0.859(124) + c$,

$\Rightarrow c = 266$
$v = (1.68 \times 10^{-7})t^4 - (1.33 \times 10^{-4})t^3 + 0.0525t^2 + 0.859t + 274$,

c When $t = 446$, $v = (1.68 \times 10^{-7})(446)^4 - (1.33 \times 10^{-4})(446)^3 + 0.0525(446)^2 + 0.859(446) + 274$
$= 5950$

When $t = 446$s, the velocity of the space shuttle is 5950 m s^{-1} (5.95 km s^{-1}).

13 d For this section of the flight:
$a = 28.6$, $u = 5950$, $v = 7850 \text{ m s}^{-1}$
$v = u + at$

$7850 = 5950 + 28.6t$
$$t = \frac{7850 - 5950}{28.6}$$
$= 66.4$

Total time to reach escape velocity = 446 + 66.4
= 510 (2 s.f.)

The shuttle cuts its main engines 510 s after launch.

Challenge

1 $v = \int a \, dt$

$= \int (3t^2 - 18t + 20) \, dt$

$= t^3 - 9t^2 + 20t + c$, where c is a constant of integration.

When $t = 0$, $v = 0$

$0 = 0 - 0 + 0 + c \Rightarrow c = 0$

$v = t^3 - 9t^2 + 20t$

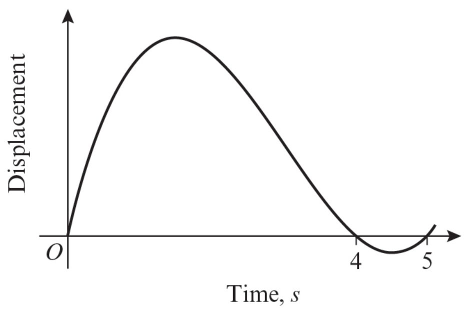
Checking for crossing points to find if the velocity becomes negative during first 5 s:

$t^3 - 9t^2 + 20t = 0$

$t(t^2 - 9t + 20) = 0$

$t(t - 4)(t - 5) = 0$

This means that displacement is positive for the first four seconds and negative in the fifth second (see sketch graph), so need to find distances separately.



s_1 = distance travelled in first 4 s

s_2 = distance travelled in fifth second

$$s_1 = \int_0^4 (t^3 - 9t^2 + 20t) \, dt$$

$$= \left[\frac{t^4}{4} - 3t^3 + 10t^2 \right]_0^4$$

$$= \left(\frac{4^4}{4} - 3 \times 4^3 + 10 \times 4^2 \right) - 0$$

$= 64 - 192 + 160$

$= 32$

$$s_2 = \int_4^5 (t^3 - 9t^2 + 20t) \, dt$$

$$= \left[\frac{t^4}{4} - 3t^3 + 10t^2 \right]_4^5$$

$$= \left(\frac{5^4}{4} - 3 \times 5^3 + 10 \times 5^2 \right) - 32$$

$$= \frac{625}{4} - 375 + 250 - 32$$

$= -0.75$

1 Total distance = 32 + 0.75 = 32.75
The particle covers 32.75 m in the first 5 s of its motion.

2 $v = \int a\,\mathrm{d}t$
$= \int (6t + 2)\,\mathrm{d}t$
$= 3t^2 + 2t + c$, where c is a constant of integration.

Assuming that velocity does not change direction during this time, distance travelled between $t = 3$ and $t = 4$

$v = \int_3^4 (3t^2 + 2t + c)\,\mathrm{d}t$
$= \left[t^3 + t^2 + ct\right]_3^4$
$= (4^3 + 4^2 + 4c) - (3^3 + 3^2 + 3c)$
$= 64 + 16 + 4c - 27 - 9 - 3c$
$= 44 + c$
So $50 = 44 + c \Rightarrow c = 6$
$v = 3t^2 + 2t + 6$

When $t = 5$, $v = 3 \times 5^2 + 2 \times 5 + 6$
$= 75 + 10 + 6$
$= 91$

At 5 s, the velocity is 91 m s^{-1}.

Review Exercise 2

1 a For the first 3 s the cyclist is moving with constant acceleration.

b For the remaining 4 s the cyclist is moving with constant speed.

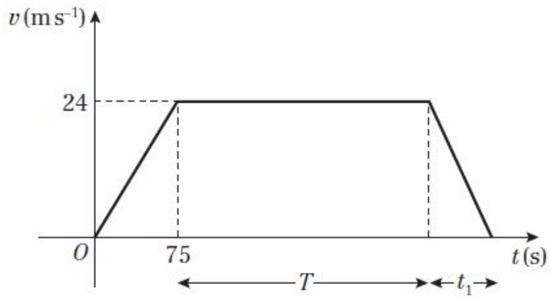
c area = trapezium + rectangle

$s = \frac{1}{2}(2+5)\times 3 + 5\times 4$

$= 10.5 + 20 = 30.5$

The distance travelled by the cyclist is 30.5 m

2 a



b Let time for which the train decelerates be t_1 s.

While decelerating

area $= \frac{1}{2}$ base × height

$600 = \frac{1}{2}t_1 \times 24$

$t_1 = \frac{1200}{24} = 50$

Acceleration is represented by the gradient.

$a = -\frac{24}{t_1} = -\frac{24}{50} = -0.48$

The deceleration is $0.48\,\mathrm{m\,s^{-2}}$

c For the whole journey

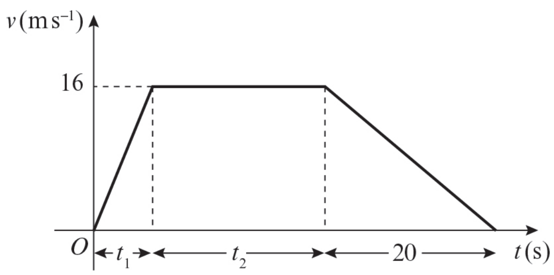
$s = \frac{1}{2}(a+b)h$

$7500 = \frac{1}{2}(T + T + 125)\times 24$

$T = \frac{7500 - 1500}{24} = 250$

d Total time is $(75 + T + t_1)\,\mathrm{s} = (75 + 250 + 50)\,\mathrm{s} = 375$ s

3 a



3 a Let the time for which the train accelerates be t_1 s and the time for which it travels at a constant speed be t_2 s.

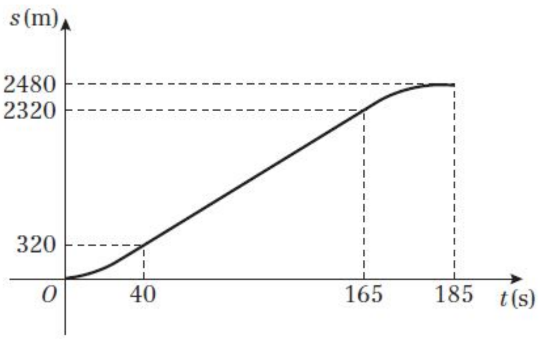
During acceleration
$v = u + at$
$16 = 0 + 0.4t_1 \Rightarrow t_1 = \frac{16}{0.4} = 40$

At constant speed
$2000 = 16 \times t_2 \Rightarrow t_2 = \frac{2000}{16} = 125$

The total time is $(t_1 + t_2 + 20)\,\text{s} = (40 + 125 + 20)\,\text{s} = 185\,\text{s}$

b $s = \frac{1}{2}(a+b)h$
$= \frac{1}{2}(125 + 185) \times 16 = 2\,480$
$AB = 2\,480\,\text{m}$

c



4 a Taking the upwards direction as positive.
$s = 40,\ v = 0,\ a = -9.8,\ u = ?$
$v^2 = u^2 + 2as$
$0^2 = u^2 - 2 \times 9.8 \times 40$
$u^2 = 784 \Rightarrow u = 28$

The speed of projection is 28 m s^{-1}

b $s = 0,\ u = 28,\ a = -9.8,\ t = ?$
$s = ut + \frac{1}{2}at^2$
$0 = 28t - 4.9t^2 = t(28 - 4.9t)$
$t = 0,\ t = \frac{28}{4.9} = 5.714...$

The time taken to return to A is 5.7 s (2 s.f.)

5 Find the speed of projection.
Taking the upwards direction as positive.

$v = 0,\ t = 3,\ a = -9.8,\ u = ?$

5 $v = u + at$

$0 = u - 9.8 \times 3 \Rightarrow u = 29.4$

$s = 39.2,\ u = 29.4,\ a = -9.8,\ t = ?$

$s = ut + \frac{1}{2}at^2$

$39.2 = 29.4t - 4.9t^2$

$4.9t^2 - 29.4t + 39.2 = 0$

Dividing all terms by 4.9

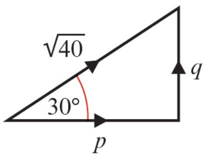
$t^2 - 6t + 8 = 0$

$(t-2)(t-4) = 0$

$t = 2, 4$

The ball is 39.2 m above its point of projection when $t = 2$ or when $t = 4$

6



$\tan 30 = \frac{1}{\sqrt{3}} = \frac{q}{p}$

$\therefore q = \frac{p}{\sqrt{3}}$

$40 = p^2 + q^2$

$p^2 + \frac{p^2}{3} = 40$

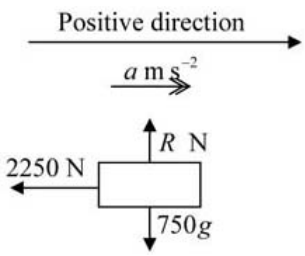
$\frac{4}{3}p^2 = 40$

$p^2 = 30$

$\therefore q^2 = 40 - 30 = 10$

The values are: $p = \sqrt{30}$ and $q = \sqrt{10}$.

7



$F = ma$

$R(\rightarrow)\ \ -2250 = 750a$

$a = -\frac{2250}{750} = -3$

7 $u = 25, v = 15, a = -3, s = ?$

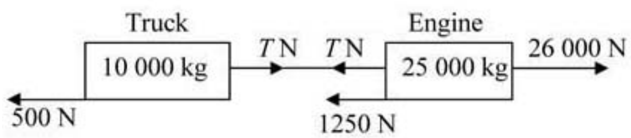
$$v^2 = u^2 + 2as$$

$$15^2 = 25^2 - 6s$$

$$s = \frac{25^2 - 15^2}{6} = \frac{400}{6} = 66\tfrac{2}{3}$$

The distance travelled by the car as its speed is reduced is $66\frac{2}{3}$ m.

8



a The resistance on the engine is $25 \times 50 = 1250$ N
The resistance on the truck is $10 \times 50 = 500$ N
For the whole system, the engine and truck

$R(\rightarrow)$ $\quad F = ma$

$$26\,000 - 1250 - 500 = 35\,000a$$

$$a = \frac{26\,000 - 1250 - 500}{35\,000} = \frac{97}{140} = 0.6928...$$

The acceleration of the engine and truck is $0.693\,\text{m s}^{-2}$ (3 s.f.)

b For the truck alone

$$F = ma$$

$$T - 500 = 10\,000a$$

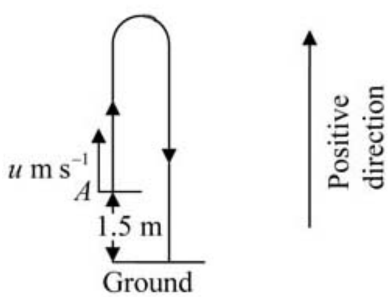
$$T = 500 + 10\,000 \times 0.6928... = 7428.57...$$

The tension in the coupling is 7430 N (3 s.f.)

c i Treating the engine and truck as particles allows us to assume the weight acts from the centre of mass of each object, and ignore wind resistance and rotational forces.

ii By assuming the coupling is a light horizontal rod, we treat it as if it had no mass and therefore can assume it not only stays straight but that it has no weight and the tension is constant along the entire length.

9



9 a From A to the greatest height, taking upwards as positive.

$v = 0,\ a = -9.8,\ s = 25.6,\ u = ?$

$v^2 = u^2 + 2as$

$0^2 = u^2 + 2 \times (-9.8) \times 25.6$

$u^2 = 2 \times 9.8 \times 25.6 = 501.76$

$u = \sqrt{501.76} = 22.4$, as required.

b $u = 22.4,\ s = -1.5,\ a = -9.8,\ t = T$

$s = ut + \frac{1}{2}at^2$

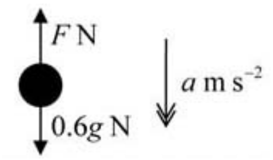
$-1.5 = 22.4T + \frac{1}{2}(-9.8)T^2$

$4.9T^2 - 22.4T - 1.5 = 0$

$$T = \frac{22.4 + \sqrt{(-22.4)^2 - 4 \times 4.9 \times -1.5}}{2 \times 4.9}$$

$= 4.637... = 4.64$ (3 s.f.)

c



To find the speed of the ball as it reaches the ground.

$u = 22.4,\ s = -1.5,\ a = -9.8,\ v = ?$

$v^2 = u^2 + 2as = 22.4^2 + 2 \times (-9.8) \times (-1.5) = 531.16$

To find the deceleration as the ball sinks into the ground.

$u^2 = 531.16,\ v = 0,\ s = 0.025,\ a = ?$

$v^2 = u^2 + 2as \Rightarrow 0^2 = 531.16 + 2 \times a \times 0.025$

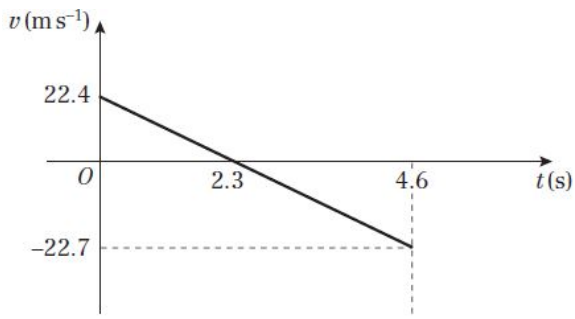
$a = -\frac{531.16}{0.05} = -10\,623.2$

$F = ma$

$0.6g - F = 0.6 \times (-10\,623.2)$

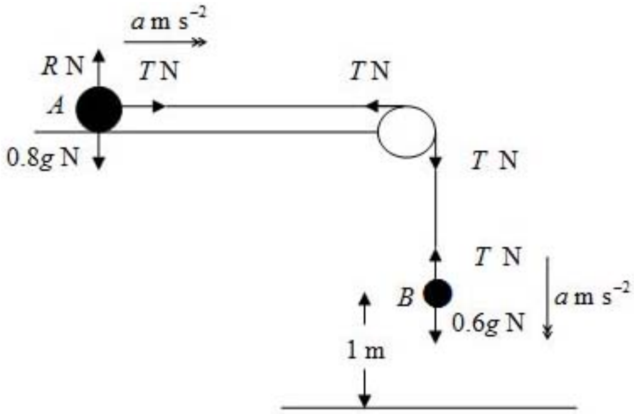
$F = 0.6g + 0.6 \times 10\,623.2 = 6380$ (3 s.f.)

d



9 e Consider air resistance during motion under gravity.

10



a For A

$R(\rightarrow)$ $T = 0.8a$ **(1)**

For B

$R(\downarrow)$ $0.6g - T = 0.6a$ **(2)**

(1) + **(2)**

$0.6g = 1.4a$

$$a = \frac{0.6 \times 9.8}{1.4} = 4.2$$

The acceleration of A is 4.2 m s^{-2}

b Substitute $a = 4.2$ into (1)

$T = 0.8 \times 4.2 = 3.36$

The tension in the string is 3.4 N (2 s.f.)

c $u = 0,\ a = 4.2,\ s = 1,\ v = ?$

$v^2 = u^2 + 2as$

$= 0^2 + 2 \times 4.2 \times 1 = 8.4$

$v = \sqrt{8.4} = 2.898...$

The speed of B when it hits the ground is 2.9 m s^{-1} (2 s.f.)

d $u = 0,\ a = 4.2,\ s = 1,\ t = ?$

$s = ut + \frac{1}{2}at^2$

$1 = 0 + 2.1t^2$

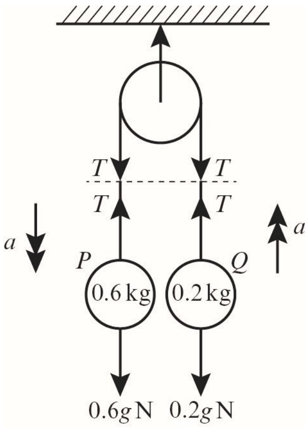
$t^2 = \frac{1}{2.1} \Rightarrow t = 0.690...$

The time taken for B to reach the ground is 0.69 s (2 s.f.)

e i Describing the string as 'light' means it has no mass (and therefore no weight).

10 e ii This fact allows us to assume that the tension is constant in all parts of the string.

11



a $F = ma$

For $P \downarrow$ positive: $0.6a = 0.6g - T$ **(1)**

For $Q \uparrow$ positive: $0.2a = T - 0.2g$ **(2)**

$3 \times$ (**2**): $0.6a = 3T - 0.6g$

Subtracting (**1**) from this: $0 = 4T - 1.2g$

$4T = 1.2g = 1.2 \times 9.8$

The tension in the string is 2.9 N (2 s.f.)

b (1) + (2): $0.8a = 0.4g$

$$a = \frac{g}{2} = \frac{9.8}{2}$$

The acceleration is 4.9 m s^{-2}.

c For P, before string breaks, taking up as positive:

$s = ?$, $u = 0$, $a = 4.9$, $t = 0.4$

$$s = ut + \tfrac{1}{2}at^2$$

$$s = (0 \times 0.4) + \tfrac{1}{2}(4.9 \times 0.4^2)$$

$$= \tfrac{1}{2}(0.784)$$

$$= 0.392\text{ m}$$

The total distance P has to fall is therefore $1 - 0.392 = 0.608$ m.

$v = u + at$

$v = 0 + (0.4 \times 4.9) = 1.96$ m s^{-1}

Before the string breaks, P is moving downwards at 1.96 m s^{-1}. After string breaks, taking down as positive, P moves under gravity.

$s = 0.608$, $u = 1.96$, $a = 9.8$, $t = ?$

$$s = ut + \tfrac{1}{2}at^2$$

$$0.608 = 1.96t + \tfrac{1}{2}(9.8 \times t^2)$$

$$0 = 4.9t^2 - 1.96t - 0.608$$

$$t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

11 c $t = \dfrac{-1.96 \pm \sqrt{(-1.96)^2 - (4 \times 4.9 \times -0.608)}}{2 \times 4.9}$

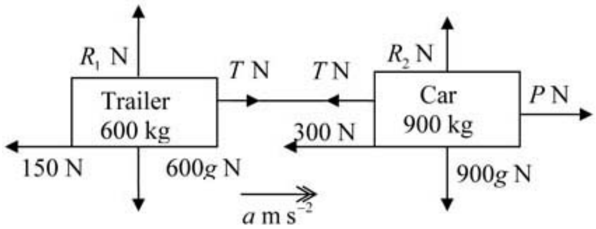
$= \dfrac{-1.96 \pm \sqrt{15.76}}{9.8}$

$= 0.205\text{ s or } -0.605\text{ s (3 d. p.)}$

Only positive answers are relevant in this context. $\therefore$ P hits the floor 0.21 s (2 s.f.) after the string breaks.

d This fact allows us to assume that the tension is constant in all parts of the string and that the acceleration of the two particles is the same.

12



a i For the whole system:

$F = ma$

$R(\rightarrow)$ $\quad P - 300 - 150 = 1500 \times 0.4$

$P = 1050$

The tractive force exerted by the engine of the car is 1050 N.

ii For the trailer alone:

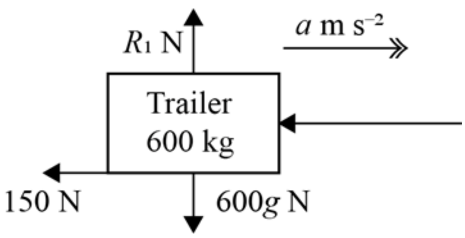
$F = ma$

$R(\rightarrow)$ $\quad T - 150 = 600 \times 0.4$

$T = 390$

The tension in the tow bar is 390 N.

b For the trailer alone:



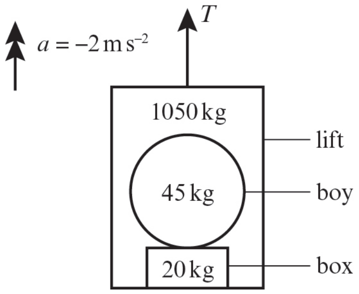
$F = ma$

$R(\rightarrow)$ $\quad 1650 + 150 = 600a$

$a = -\frac{1800}{600} = -3$

The greatest possible deceleration of the car is 3 m s^{-2}

13

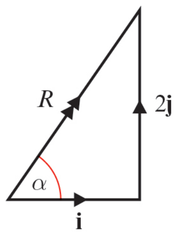


a $F = ma$
Taking up as positive:
$(1050 + 45 + 20) \times -2 = T - (1050 + 45 + 20)g$
$T = 1115(g - 2)$
$T = 1115 \times 7.8$
The tension in the cable is 8697 N.

b From Newton's third law of motion:
|Force exerted on boy by box| = |Force exerted on box by boy| = $|R_1|$
For the boy, taking up as positive:
$45 \times -2 = R_1 - 45g$
$R_1 = 45(g - 2)$
$R_1 = 45 \times 7.8$
The boy exerts a force of 351 N on the box.

c From Newton's third law of motion:
|Force exerted on box by lift| = |Force exerted on lift by box| = $|R_2|$
For the box, taking up as positive:
$20 \times -2 = R_2 - 20g - 351$
$R_2 = 351 + 20(g - 2)$
$R_2 = 351 + (20 \times 7.8) = 351 + 156$
The box exerts a force of 507 N on the lift.

14 a



Let the required angle be α.
Then $\tan\alpha = 2$
$\therefore \alpha = 63^\circ\,(2\text{ s.f.})$

b As $\mathbf{F}_1 + \mathbf{F}_2 = \mathbf{R}$
$(2\mathbf{i} + 3\mathbf{j}) + (\lambda\mathbf{i} + \mu\mathbf{j}) = k(\mathbf{i} + 2\mathbf{j})$

where k is a constant.
$\therefore 2 + \lambda = k$ and $3 + \mu = 2k$ *

Eliminate k from these two equations.

14 b Then $2(2+\lambda)=3+\mu$

$\therefore 2\lambda-\mu+1=0$

c If $\mathbf{F}_2$ is parallel to $\mathbf{j}$ then $\lambda=0$

Substituting $\lambda=0$ into * gives

$\mu=1$ and $k=2$

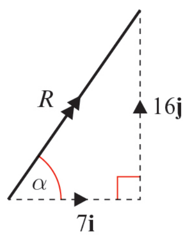
$\therefore \mathbf{R}=2\mathbf{i}+4\mathbf{j}$

$\therefore |\mathbf{R}|=\sqrt{2^2+4^2}$

$=\sqrt{20}$

$=4.47$ (3 s.f.)

15



a The magnitude of

$\mathbf{R}=\sqrt{7^2+16^2}$

$=17.5$ (1 d.p.)

b $\tan\alpha=\frac{16}{7}$

$\alpha=\tan^{-1}\left(\frac{16}{7}\right)$

$=66^\circ$ (nearest degree)

c Let $\mathbf{P}=\lambda(\mathbf{i}+4\mathbf{j})$ and $\mathbf{Q}=\mu(\mathbf{i}+\mathbf{j})$

As $\mathbf{P}+\mathbf{Q}=\mathbf{R}$

$\therefore \lambda(\mathbf{i}+4\mathbf{j})+\mu(\mathbf{i}+\mathbf{j})=(7\mathbf{i}+16\mathbf{j})$

Equating $\mathbf{i}$ components

$\lambda+\mu=7$ **(1)**

Equating $\mathbf{j}$ components

$4\lambda+\mu=16$ **(2)**

Subtract **(2)** – **(1)**

$3\lambda=9$

$\therefore \lambda=3$

15 Substitute into equation (**1**)

$\therefore 3 + \mu = 7$

$\therefore \mu = 4$

$\therefore \mathbf{P} = 3(\mathbf{i} + 4\mathbf{j}) = 3\mathbf{i} + 12\mathbf{j}$ and $\mathbf{Q} = 4(\mathbf{i} + \mathbf{j}) = 4\mathbf{i} + 4\mathbf{j}$

16 $a = 5 - 2t$

$v = \int a\text{d}t = \int (5 - 2t)\text{d}t$

$= 5t - t^2 + C$

When $t = 0,\ v = 6$

$6 = 0 - 0 + C \Rightarrow C = 6$

Hence

$v = 6 + 5t - t^2$

When P is at rest

$0 = 6 + 5t - t^2$

$t^2 - 5t - 6 = (t - 6)(t + 1) = 0$

$t = 6, -1$

$t > 0$

$\therefore t = 6$

P is at rest at $t = 6$ s

17 $v = 6t - 2t^2$

a Maximum value of velocity occurs when $a = 0$

$a = \dfrac{dv}{dt} = 6 - 4t$

Maximum velocity occurs at $t = \dfrac{3}{2}$ s

$v = \left(6 \times \dfrac{3}{2}\right) - 2\left(\dfrac{3}{2}\right)^2$

$v = 9 - \dfrac{9}{2} = \dfrac{9}{2}$

The maximum velocity is 4.5 m s^{-1}.

b When P returns to O, $s = 0$

$s = \int v dt = \int 6t - 2t^2 dt$

$s = 3t^2 - \dfrac{2}{3}t^3 + c$

At $t = 0$, $s = 0$ so $c = 0$

17 $0 = t^2\left(3 - \frac{2}{3}t\right)$

$t = 0$ or $\frac{2}{3}t = 3$

P returns to O after 4.5 s.

18 $v = 3t^2 - 8t + 5$

a When the particle is at rest, $v = 0$

$0 = 3t^2 - 8t + 5$

$0 = 3\left(t^2 - \frac{8}{3}t + \frac{5}{3}\right)$

$0 = 3\left(t - \frac{3}{3}\right)\left(t - \frac{5}{3}\right)$

(or by using quadratic equation formula)

P is at rest at 1 s and $\frac{5}{3}$ s.

b $a = \frac{dv}{dt} = \frac{d}{dt}(3t^2 - 8t + 5)$

$a = 6t - 8$

$t = 4$

$a = (6 \times 4) - 8$

After 4 s, the acceleration of P is 16 m s^{-2}.

c Distance travelled in third second $= s_3$

$s_3 = \int_2^3 v\,dt = \int_2^3 3t^2 - 8t + 5\,dt$

$s_3 = \left[t^3 - 4t^2 + 5t\right]_2^3$

$s_3 = [27 - 36 + 15] - [8 - 16 + 10]$

$s_3 = 6 - 2$

The distance travelled in the third second is 4 m.

19 $v = 6t - 2t^{\frac{3}{2}}$

a $a = \frac{dv}{dt}$

$a = 6 - 3t^{\frac{1}{2}}$

b At $t = 0$, $s = 0$

$s = \int v\,dt = \int 6t - 2t^{\frac{3}{2}}\,dt$

$s = 3t^2 - \frac{4}{5}t^{\frac{5}{2}} + c$

At $t = 0$, $s = 0$, $c = 0$

So $s = 3t^2 - \frac{4}{5}t^{\frac{5}{2}}$

Challenge

1 $t_1 + t_2 + t_3 = 7 \times 60 = 420$
$3t_1 = 4t_3$
$t_3 = 0.75t_1$

Considering time t_1

$$s = \left(\frac{u+v}{2}\right)t_1$$

$$1750 = \left(\frac{0+v}{2}\right)t_1$$

$$v = \frac{3500}{t_1}$$

Considering time t_2
$s_2 = vt_2$

$$17\,500 = \frac{3500}{t_1}t_2$$

$$t_2 = 5t_1$$

Considering total time:
$t_1 + 5t_1 + 0.75\, t_1 = 420$

$$t_1 = \frac{420}{6.75} = 62.22 \text{ s}$$

$\therefore t_2 = 311.11$ s
& $t_3 = 46.67$ s

Distance travelled during time t_3 is s_3

$$s_3 = \left(\frac{u+v}{2}\right)t_3$$

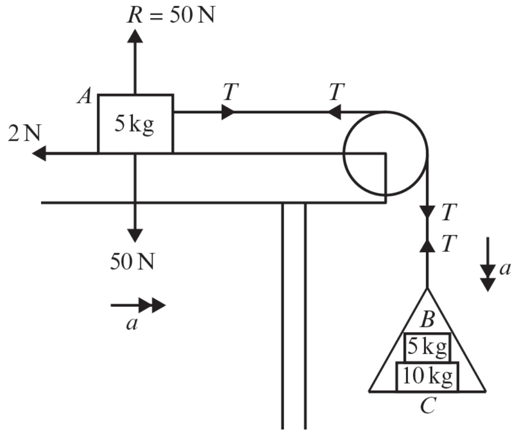
$$u = \frac{3500}{t_1} = \frac{3500}{62.22} = 56.25,\ v = 0,\ t = 46.67$$

$$s_3 = \left(\frac{56.25+0}{2}\right)46.67$$

$$s_3 = 28.125 \times 46.67 = 1312.6$$

Total distance = 1750 + 17 500 + 1312.6
The distance between the two stations is 20.6 km (3 s.f.).

2



a Considering A, → positive: $T - 2 = 5a$
Considering entire pan, ↓ positive: $(5 + 10)g - T = 15a$
So $150 - T = 15a$

Adding these gives:
$148 = 20a$
The acceleration of the pan is 7.4 m s^{-2}.

b Substituting this value into the first equation gives:
$T - 2 = 5 \times 7.4 = 37$
The tension in the string is 39 N.

c Block C exerts a normal reaction force R on block B.
Considering block B only, ↓ positive:
$5g - R = 5a$
$50 - R = 37$
Block C exerts a force of 13 N on block B.

d The force the string exerts on the pulley has two perpendicular components, each of magnitude T.
The magnitude of the total force, F, is therefore given by:
$F^2 = T^2 + T^2$
$F = \sqrt{39^2 + 39^2} = \sqrt{3042}$
The string exerts a force of magnitude 55 N (2 s.f.) on the pulley.

e The fact that the string is inextensible allows us to assume that the tension is constant in all parts of the string and that the acceleration of Block A and the scale pan are the same.

Exam–style Practice paper

Section A: Statistics

1 a $0.2 + y + 0.3 + 0.35 = 1$
$y = 1 - 0.85 = 0.15$

b $P(B \text{ and } M) = 0.15$
$P(B) \times P(M) = (0.2 + 0.15) \times (0.3 + 0.15) = 0.35 \times 0.45 = 0.1575$
$P(B \text{ and } M) \neq P(B) \times P(M)$, so 'likes bananas' and 'likes mangoes' are not independent events.

2 a t is a continuous variable, because it is a measured variable which can take any value.

b mean $= \dfrac{\sum t}{n} = \dfrac{140.1}{10} = 14.01$

standard deviation $= \sqrt{\dfrac{\sum t^2}{n} - \left(\dfrac{\sum t}{n}\right)^2} = \sqrt{\dfrac{1981.33}{10} - \left(\dfrac{140.1}{10}\right)^2} = 1.36$ (to 3 s.f.)

c 15.8 °C is higher than the current mean so the mean would increase.

d Clare could take a random sample of days from the whole of September for the different locations in the UK.

3 a $0.1 + 0.2 + 0.15 + p + 0.1 + 0.25 = 1$
$p = 1 - 0.8 = 0.2$

b $P(2 \leq X \leq 5) = 1 - P(X = 1) - P(X = 6)$
$= 1 - 0.1 - 0.25 = 0.65$

c i $P(\text{odd}) = 0.1 + 0.15 + 0.1 = 0.35$

$P(\text{odd exactly twice}) = \binom{10}{2} 0.35^2 0.65^8$
$= 0.1757$ (to 4 d.p.)

ii $P(\text{odd more than 6 times}) = \binom{10}{7} 0.35^7 0.65^3 + \binom{10}{8} 0.35^8 0.65^2 + \binom{10}{9} 0.35^9 0.65^1 + 0.35^{10}$
$= 0.0260$ (to 4 d.p.)

4 a The test statistic is the number of plates that are flawed.
$H_0: p = 0.3$, $H_1: p < 0.3$

b $X \sim B(20, 0.3)$
$P(X \leq 2) = 0.0355 < 0.05$
$P(X \leq 3) = 0.1071 > 0.05$
The critical region is $X \leq 2$

c The actual significance level is 0.0355 = 3.55%

d 1 falls into the critical region, therefore there is evidence to support the claim.

5 a The increase in energy released is 3.1 Joules for each degree of temperature.

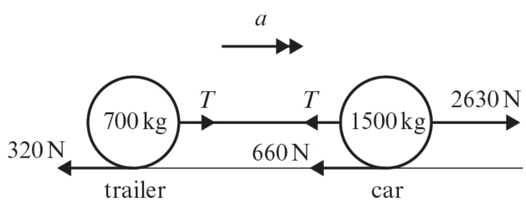
5 b This value of h is a long way from the range of the experimental data: hence the extrapolation is excessive and the predicted value of e would be too unreliable.

c It is not sensible. The regression line predicts a value of e given h, not the other way round.

6 $$P(4.6 \le h \le 6.1) = \frac{0.4\times10+0.2\times45+0.2\times60+0.2\times80+0.4\times25+0.1\times10}{0.5\times5+0.5\times10+0.2\times45+0.2\times60+0.2\times80+0.4\times25+0.5\times10}$$
$$= \frac{4+9+12+16+10+1}{2.5+5+9+12+16+10+5}$$
$$= \frac{52}{59.5}$$
$= 0.87$ (to 2 d.p.)

Section B: Mechanics

7



$F = ma$

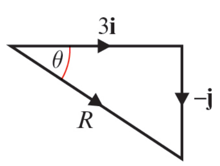
a For the whole system:
$F = 2630 - 660 - 320 = 1650$
$m = 1500 + 700 = 2200$
$1650 = 2200a$
The acceleration of the car is 0.75 m s^{-2}

b For the trailer:
$F = T - 320$, $m = 700$, $a = 0.75$
$T - 320 = 700 \times 0.75 = 525$
$T = 525 + 320$
The tension in the tow-rope is 845 N.

c Since the tow-rope is inextensible, the acceleration of each part of the system is identical and the tension in it is constant throughout.

8 a Resultant force, $F = \mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3$
$F = (3\mathbf{i} - 6\mathbf{j}) + (4\mathbf{i} + 5\mathbf{j}) + (2\mathbf{i} - 2\mathbf{j}) = (9\mathbf{i} - 3\mathbf{j})$
$m = 3$, $\mathbf{a} = ?$
$F = m\mathbf{a}$
$(9\mathbf{i} - 3\mathbf{j}) = 3\mathbf{a}$
The acceleration of the particle is $(3\mathbf{i} - \mathbf{j})$ m s^{-2}

b



$\tan\theta = \frac{1}{3}$

8 b The acceleration acts at an angle of 18.4° below **i**.

c $|\mathbf{a}| = \sqrt{1^2 + 3^2} = \sqrt{10}$

The magnitude of the acceleration is $\sqrt{10}$ m s^{-2}

9 a Taking up as positive:

$s = 0$, $a = -9.8$, $t = 5$, $u = ?$

$s = ut + \frac{1}{2}at^2$

$0 = (u \times 5) + \frac{1}{2}(-9.8 \times 5^2) = 5u - 122.5$

$u = \frac{122.5}{5} = 24.5$

The ball is projected at a speed of 24.5 m s^{-1}

b $u = 24.5$, $a = -9.8$, $v = 0$, $s = ?$

$v^2 = u^2 + 2as$

$0 = 24.5^2 + (2 \times (-9.8s))$

$s = \frac{24.5^2}{2 \times 9.8} = \frac{600.25}{19.6} = 30.625$

The ball reaches a height of 30.6 m above P.

c $s = 15$, $u = 24.5$, $a = -9.8$, $t = ?$

$s = ut + \frac{1}{2}at^2$

$15 = 24.5t + \frac{1}{2}(-9.8 \times t^2)$

$4.9t^2 - 24.5t + 15 = 0$

$t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$t = \frac{24.5 \pm \sqrt{24.5^2 - (4 \times 4.9 \times 15)}}{2 \times 4.9}$

$= \frac{24.5 \pm \sqrt{306.25}}{9.8}$

$= \frac{24.5 \pm 17.5}{9.8}$

$= \frac{42}{9.8}$ or $\frac{7}{9.8}$

The ball is at a height of 15 m above P at 0.714 s and 4.29 s after leaving P.

10 $v = 3 + 9t^2 - 4t^3$

When the particle is moving at maximum velocity, $a = \frac{dv}{dt} = 0$

$0 = \frac{d\left(3 + 9t^2 - 4t^3\right)}{dt}$

$= 18t - 12t^2$

$= 6t(3 - 2t)$

10 At $t = 0$, the particle moves at minimum velocity (see graph).
The particle has maximum velocity at $t = \frac{3}{2}$ seconds.

$$s = \int v \, dt = \int_0^{\frac{3}{2}} \left(3 + 9t^2 - 4t^3\right) dt$$

$$= \left[3t + \frac{9t^3}{3} - \frac{4t^4}{4}\right]_0^{\frac{3}{2}} = \left[3t + 3t^3 - t^4\right]_0^{\frac{3}{2}}$$

For $t = 0$, all terms are zero, so this becomes:

$$s = 3 \times \left(\frac{3}{2}\right) + 3 \times \left(\frac{3}{2}\right)^3 - \left(\frac{3}{2}\right)^4$$

$$= \frac{9}{2} + \frac{81}{8} - \frac{81}{16} = \frac{153}{16}$$

The particle is moving at maximum velocity when it is $\frac{153}{16}$ m from O.